

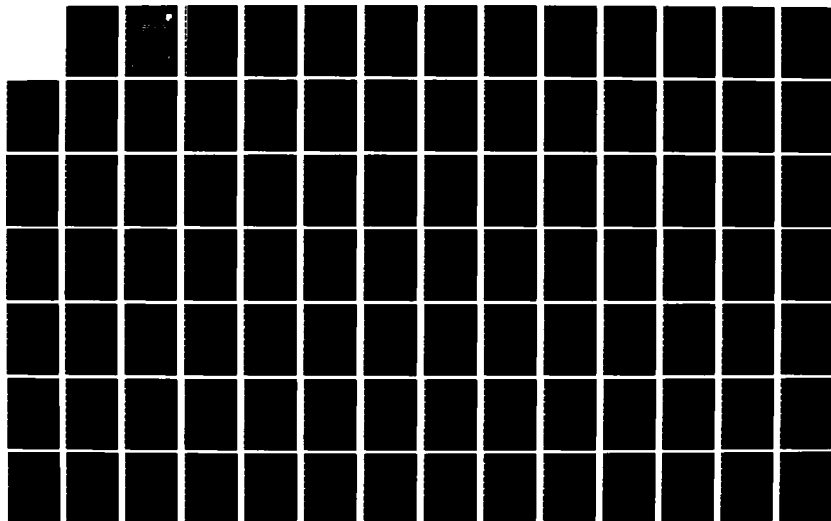
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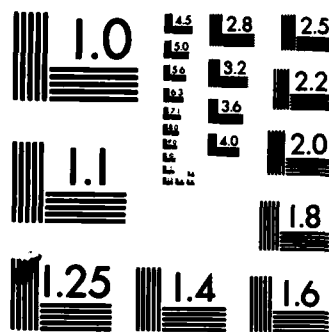
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Interim Report

November 1983



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SCATTERING AND DEPOLARIZATION OF ELECTROMAGNETIC WAVES-FULL WAVE SOLUTIONS

University of Nebraska-Lincoln

Ezekiel Bahar

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In the first Annual Technical Report (1 Mar 81 - 28 Feb 82) the principal elements of the full wave approach and its relationship to earlier solutions of scattering problems were summarized. For the convenience of the reader of this report, all the principal analytical results leading to full wave expressions for the normalized cross sections are presented in Section (1.1). Thus, starting with Maxwell's equations for the transverse components of the electromagnetic fields, the boundary conditions, the expressions for the com- plete field expansions and the associated orthogonal relations, the generalized		

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telegraphists equations are derived. It is shown how these coupled equations are solved using a second order iterative approach and the judicious use of a local coordinate system that conforms with features of the rough surface. Explicit expressions for the scattered radiation fields and the like and cross polarized scattering cross sections are also presented. Thus the reader finds in this section a convenient review of all the background material that leads up to the general full wave expressions for the scattering cross sections.

It was shown that the stationary phase approximation of the full wave solution is precisely equal to the physical optics solution for scattering by rough surfaces. Thus, if the principal contributions to the scattered fields come from the regions around specular points of the rough surface, at high frequencies, the physical optics approximations are valid. On the other hand, if the principal contributions to the scattered fields do not come from specular points of the rough surface, the physical optics solutions are not valid even if the surface meets the radii of curvature criteria associated with the Kirchhoff approximations for the surface fields. Similarly it was shown that if the scale of the surface roughness and the slopes are small, the full wave solutions reduce precisely to the perturbation solution for the scattered fields. Thus the full wave solutions which account for both specular point scattering as well as Bragg scattering in a uniform self-consistent manner, resolve the apparent discrepancies between the physical optics and perturbation theories. Since at near normal incidence the backscattered fields are primarily due to specular point scattering while the backscattered fields near grazing incidence are primarily due to Bragg scattering, a two-scale model "mostly based on physical considerations" was advanced. These composite models are able to explain features in radar cross section that no theory can. A problem which is directly associated with the two-scale model of the composite rough surface concerns the specification of the wavenumber k_d where spectral splitting between the large and small scale surfaces is assumed to occur. Furthermore, the two-scale model of the composite surface is restricted by the assumption that the large and small surface heights h_L and h_s are statistically independent. In order to examine the earlier results based on the two-scale models of composite surfaces, the full wave approach (which accounts for both specular point scattering as well as Bragg scattering in a uniform self-consistent manner) is also applied to the two-scale model. Thus in Sections 2 and 3 of this report the composite surface is decomposed into a large and small scale surface.

In Section 4 the full wave approach is used to determine the scattering cross sections without assuming the two-scale model of the surface. Thus in this case the problems associated with the specifications of k_d do not arise and it is necessary to assume that the large and small scale surfaces are statistically independent. Using the full wave approach specular point scattering and Bragg scattering are not artificially separated out in the expressions for the scattering cross sections.

It is shown that while the values for the like polarized backscatter cross sections obtained from the unified and two-scale versions of the full wave solutions are in substantial agreement for all angles of incidence, there are very significant differences between the values of the cross polarized backscatter cross sections obtained from the uniform and two-scale models of the rough surface. This is shown to be primarily because the physical optics solutions for the cross-polarized backscatter cross sections vanish. However, as is shown through the use of the unified full wave approach, only at the specular points of the rough surface is there no depolarization of the backscattered fields. Thus a two-scale model based on a perturbed-physical optics approach should not be used to determine the cross polarized scattering cross sections.

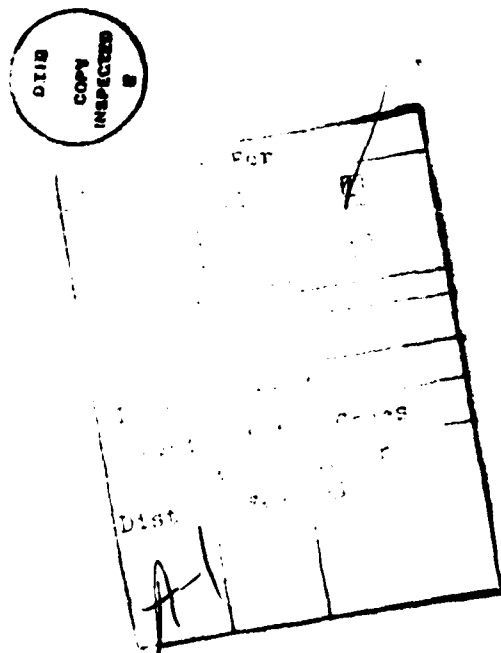
In Section 5 a summary of the proposed research for the next six months is presented and suggestions for future investigations are also listed.

TABLE OF CONTENTS

	<u>Page</u>
1.0 INTRODUCTION	1
1.1 Summary of Research	7
1.2 Interim Technical Reports and Publications in Scientific/Technical Journals	28
2.0 SCATTERING CROSS SECTIONS FOR COMPOSITE SURFACES THAT CANNOT BE TREATED AS PERTURBED-PHYSICAL OPTICS PROBLEMS	30
2.1 Background	30
2.2 Discussion	30
2.3 Formulation of the Problem	34
2.4 Application to Surfaces That Do Not Satisfy Perturbation and Physical Optics Criteria	43
2.5 Concluding Remarks	52
3.0 COMPUTATIONS OF SCATTERING CROSS SECTIONS FOR COMPOSITE SURFACES AND THE SPECIFICATION OF THE WAVENUMBER WHERE SPECTRAL SPLITTING OCCURS	53
3.1 Background	53
3.2 Discussion	53
3.3 Formulation of the Problem	56
3.4 Illustrative Examples	66
3.5 Concluding Remarks	98
4.0 COMPUTATIONS OF ROUGH SURFACE CROSS SECTIONS THAT DO NOT INVOLVE SPECTRAL SPLITTING	101
4.1 Background	101
4.2 Discussion	102
4.3 Application of the Full Wave Solution Without Surface Decomposition	103
4.4 Illustrative Examples	110
4.5 Concluding Remarks	117

TABLE OF CONTENTS (continued)

	<u>Page</u>
5.0 CONCLUDING REMARKS - SUMMARY OF RESEARCH DURING THE NEXT SIX MONTHS - PROPOSED FUTURE INVESTIGATIONS . . .	119
6.0 LIST OF REFERENCES	125



1.0 INTRODUCTION

In the first Annual Technical Report (March 1, 1981-February 28, 1982), the principal elements of the full wave approach and its relationship to earlier solutions of scattering problems were summarized.

It was shown that the stationary phase approximation of the full wave solution is precisely equal to the physical optics solution for scattering by rough surfaces. Thus if the principal contributions to the scattered fields come from the regions around specular points of the rough surface, at high frequencies, $k_0^2 \langle h^2 \rangle \gg 1$, (k_0 -electromagnetic wave number, $\langle h^2 \rangle$ -mean square height), the physical optics approximations are valid. On the other hand if the principal contributions to the scattered fields do not come from specular points of the rough surface, the physical optics solutions are not valid even if the surface meets the radii of curvature criteria associated with the Kirchhoff approximations for the surface fields.

Similarly it was shown that if the scale of the surface roughness is small ($k_0^2 \langle h^2 \rangle \ll 1$) and the slopes of the rough surface are small, the full wave solutions reduce precisely to the perturbation solution for the scattered fields (Rice, 1951).

Thus the full wave solutions which account for both specular point scattering as well as Bragg scattering in a uniform self-consistent manner resolve the apparent discrepancies between the physical optics and perturbation theories.

In an attempt to account for the fact that at near normal incidence the backscattered fields are primarily due to specular point scattering while the backscattered fields near grazing incidence are primarily due to Bragg scattering, a two-scale model "mostly based on physical considerations" was advanced (Wright, 1968; Valenzuela, 1968). These composite models are

able to explain features in radar cross section that no theory can. More recently Brown (1978) used a combination of Burrows' perturbation theory and physical optics to show that the scattering cross sections for rough surfaces can be expressed as a sum of two cross sections; one associated with the filtered surface consisting of the large scale spectral components of the surface and the second associated with the surface consisting of the small scale spectral components. The results of Valenzuela (based on physical considerations) and those of Brown (based on a perturbed physical optics approach) are in agreement provided that the mean square slope of the rough surface is small.

On applying the full wave approach to the two-scale model of the composite surface it was shown that the scattering cross sections can be expressed as a weighted sum of two cross sections. It was also shown that the difference between Valenzuela's solution and Brown's solution for the scattering cross sections is primarily due to the fact that in Valenzuela's work the small scale surface height is measured perpendicular to the large scale surface and that the correlation distance for the small scale surface depends on distances measured along the large scale surface. On the other hand in Brown's work the small scale surface height is measured perpendicular to the mean, reference surface and the correlation distance is measured in the mean, reference plane. This is contrary to Burrows' perturbation formulation upon which Brown's analysis is based.

A problem which is directly associated with the two-scale model of the composite rough surface concerns the specification of the wavenumber k_d where spectral splitting between the large and small scale surfaces is assumed to occur (Brown, 1978). Using Brown's approach k_d is specified

on the basis of the features of the small scale surface h_s ; namely $\beta = 4k_o^2 \langle h_s^2 \rangle$. It is shown that Brown's results for the scattering cross sections critically depend on the value chosen for β . Since, using Brown's theory, the small scale surface must satisfy the perturbation criteria, Brown suggests that k_d should be chosen subject to the condition $\beta = 0.1$. On the other hand Hagfors (1966) and Tyler (1976) suggest that for near normal incidence the specification of k_d should be based on the characteristics of the large scale surface since the large scale surface must satisfy the radii of curvature criteria associated with the Kirchhoff approximations of the surface fields.

In addition to the above problems associated with the specification of k_d , it should be noted that the two-scale model of the composite surface is restricted by the assumption that the large and small surface heights h_l and h_s are statistically independent (Brown, 1978). In order to examine the earlier results based on the two-scale models of composite surfaces the full wave approach (which accounts for both specular point scattering as well as Bragg scattering in a uniform self-consistent manner) is also applied to the two-scale model. Thus in Sections 2 and 3 of this report the composite surface is decomposed into a large and small scale surface. However, the value of β (which directly specifies k_d) is allowed to vary from 0.1 to 2.0 since use of the full wave approach imposes no restriction on the specification of k_d . Thus assuming that the surface height spectral density function (or its Fourier transform the surface height autocorrelation function) is known, the value of k_d (where spectral splitting is assumed to occur) is determined by its relationship to

$\beta = 4k_0^2 \langle h_s^2 \rangle$ (see Section 3). The portion of the spectral density function $W(k)$ corresponding to the filtered large scale surface h_F ($k \leq k_d$) is used to determine the mean square slope of the large scale surface h_F . This in turn determines the probability density function for the large scale surface slopes since the random surface height is assumed to comprise a superposition of a sufficiently large number of zero mean independent component heights such that the surface height and all of its derivatives are Gaussian (Brown 1978). Thus all the data needed to compute the scattering cross sections (based on the two-scale model) can be obtained from the surface height spectral density function or the corresponding surface height autocorrelation function (see Sections 2 and 3). It is shown that for values of β between one and two the scattering cross sections based on the full wave solutions are practically insensitive to variations in k_d . For these values of β the filtered surface h_F satisfies the radii of curvature criteria as well as the criteria for deep phase modulation (see Sections 2 and 3). However, the perturbed-physical optics approach cannot be used for $\beta \geq 0.1$ (Brown, 1978).

In Section 4 the full wave approach is used to determine the scattering cross sections without assuming the two-scale model of the surface. Thus in this case the problems associated with the specifications of k_d do not arise and it is not necessary to assume that the large and small scale surfaces are statistically independent (Brown, 1978). Thus using the full wave approach specular point scattering and Bragg scattering are not artificially separated out in the expressions for the scattering cross sections.

It is shown in Section 4 that while the values for the like polarized backscatter cross sections ($\langle \sigma^{VV} \rangle$ and $\langle \sigma^{HH} \rangle$) obtained from the unified and two scale versions of the full wave solutions are in substantial agreement for all angles of incidence, there are very significant differences between the values of the cross polarized backscatter cross sections obtained from the unified and two-scale models of the rough surface. This is shown to be primarily because the physical optics solutions for the cross-polarized backscatter cross sections (associated with the filtered surface h_F) vanish (see Section 4). However, as is shown through the use of the unified full wave approach, only at the specular points of the rough surface is there depolarization of the backscattered fields. Thus no matter how one chooses k_d (the wavenumber where spectral splitting is assumed to occur) a two scale model based on a perturbed-physical optics approach should not be used to determine the cross polarized scattering cross sections.

For the convenience of the reader of this report, all the principal analytical results leading to full wave expressions for the normalized cross sections are presented in Section (1.1). Thus, starting with Maxwell's equations for the transverse components of the electromagnetic fields, the boundary conditions, the expressions for the complete field expansions and the associated orthogonal relations, the generalized telegraphists equations are derived. It is shown how these coupled equations are solved using a second order iterative approach and the judicious use of a local coordinate system that conforms with features of the rough surface. Explicit expressions for the scattered radiation fields and the like and cross polarized scattering cross sections are also presented in this section.

Thus the reader finds in this section a convenient review of all the background material that leads up to the general full wave expressions for the scattering cross sections.

In Section (1.2) the technical reports and publications resulting from this Air Force Contract are listed. In Section 5 a summary of the proposed research for the next six months is presented and suggestions for future investigations are also listed.

1.1 Summary of Research

In this section the principal steps leading to the derivation of the full wave solutions for the scattering cross sections for random rough surfaces are summarized. The complex electromagnetic parameters of the media above and below the irregular boundary $y = h(x)$ are (see Fig. 1.1

$$\epsilon = \epsilon_R - i\epsilon_I = \epsilon_R - \frac{i\sigma}{\omega} = \begin{cases} \epsilon_0, & y > h(x) \\ \epsilon_1, & y < h(x) \end{cases}, \quad \mu = \begin{cases} \mu_0 \\ \mu_1 \end{cases} \quad (1.1)$$

in which ϵ_R , ϵ_I , and the conductivity σ are real and an $\exp(i\omega t)$ time dependence is assumed.

In this work it is assumed that the complex permittivity ϵ and permeability μ are not functions of x . Thus wave scattering is only due to the irregular boundary between the two half spaces $y > h(x)$ (medium 0) and $y < h(x)$ (medium 1). Since it is convenient to represent electric current loops by magnetic dipoles, both electric and magnetic sources \vec{J} and \vec{M} are assumed to be present. Maxwell's equations for the transverse electric and magnetic fields, \vec{E}_T and \vec{H}_T respectively, are (Bahar 1973a,b)

$$\begin{aligned} -\frac{\partial \vec{E}_T}{\partial x} &= i\omega\mu(\vec{H}_T \times \vec{a}_x) - \frac{1}{i\omega\epsilon} \nabla_T \nabla_T \cdot (\vec{H}_T \times \vec{a}_x) \\ &\quad + \vec{M}_T \times \vec{a}_x + \frac{1}{i\omega\epsilon} \nabla_T J_x \end{aligned} \quad (1.2)$$

and

$$\begin{aligned} -\frac{\partial \vec{H}_T}{\partial x} &= i\omega\epsilon(\vec{a}_x \times \vec{E}_T) - \frac{1}{i\omega\mu} \nabla_T \nabla_T \cdot (\vec{a}_x \times \vec{E}_T) \\ &\quad + \vec{a}_x \times \vec{J}_T + \frac{1}{i\omega\mu} \nabla_T M_x, \end{aligned} \quad (1.3)$$

in which the operator ∇_T is given by

$$\nabla_T = \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z} \quad (1.4)$$

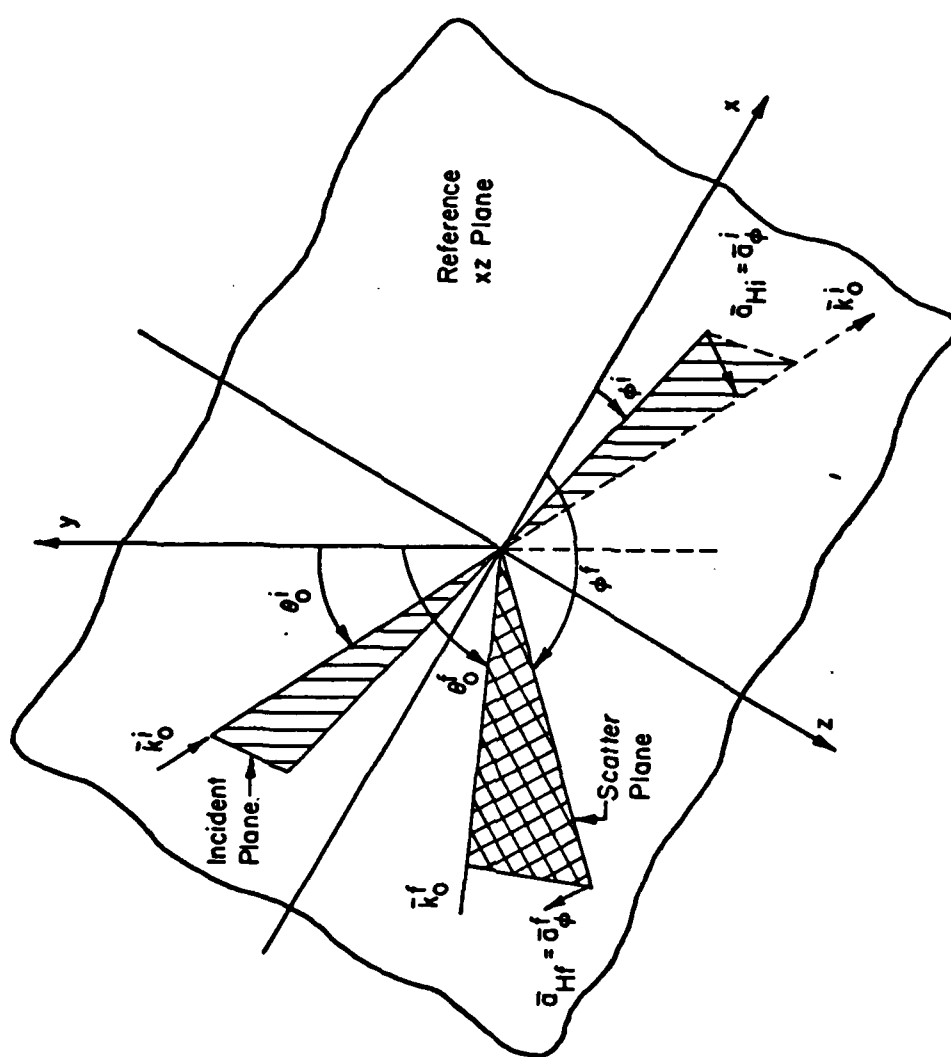


Fig. 1.1. Planes of incidence and scatter with respect to the reference coordinate system. Mean (reference) plane for rough surface is $y = 0$.

and the transverse vectors are

$$\bar{A}_T = \bar{a}_y A_y + \bar{a}_z A_z, \quad \bar{A} = \bar{E}, \bar{H}, \bar{J} \text{ or } \bar{M} \quad (1.5)$$

The following field transform pairs provide the basis for the complete expansion of the transverse electric and magnetic fields into vertically and horizontally polarized waves:

$$\bar{E}_T(x, y, z) = \sum_V \int_{-\infty}^{\infty} [E^V(x, v, w) \bar{e}_T^V + E^H(x, v, w) \bar{e}_T^H] dw, \quad (1.6)$$

where

$$E^P(x, v, w) = \int_{-\infty}^{\infty} \bar{E}_T(x, y, z) \cdot (\bar{h}_P^T \cdot \bar{a}_x) dy dz, \quad P=V \text{ or } H, \quad (1.7)$$

$$\bar{H}_T(x, y, z) = \sum_V \int_{-\infty}^{\infty} [H^V(x, v, w) \bar{h}_T^V + H^H(x, v, w) \bar{h}_T^H] dw, \quad (1.8)$$

where

$$H^P(x, v, w) = \int_{-\infty}^{\infty} \bar{H}_T(x, y, z) \cdot (\bar{a}_x \times \bar{e}_P^T) dy dz, \quad P=V \text{ or } H, \quad (1.9)$$

The basis functions for the vertically polarized waves are

$$\bar{e}_T^V = Z^V (\bar{a}_y \psi^V(v, y) - \frac{\bar{a}_z i w}{u^2 + w^2} \frac{\partial \psi^V(v, y)}{\partial y}) \phi(w, z) \quad (1.10)$$

and

$$\bar{h}_T^V = \bar{a}_z \psi^V(v, y) \phi(w, z) \quad (1.11)$$

and the complementary basis functions for the vertically polarized waves

are

$$\bar{e}_V^T = Z^V N^V (\bar{a}_y \psi^V(v, y) + \frac{\bar{a}_z i w}{u^2 + w^2} \frac{\partial \psi^V(v, y)}{\partial y}) \phi^C(w, z) \quad (1.12)$$

and

$$\bar{h}_V^T = \bar{a}_z N^V \psi^V(v, y) \phi^C(w, z). \quad (1.13)$$

For the horizontally polarized waves, the basis functions and the complementary basis functions are respectively

$$\bar{e}_T^H = \bar{a}_z \psi^H(v, y) \phi(w, z), \quad (1.14)$$

$$\bar{h}_T^H = Y^H(-\bar{a}_y \psi^H(v, y) + \frac{\bar{a}_z i w}{u^2 + w^2} \frac{\partial \psi^H(v, y)}{\partial y}) \phi(w, z), \quad (1.15)$$

and

$$\bar{e}_H^T = \bar{a}_z N^H \psi^H(v, y) \phi^c(w, z), \quad (1.16)$$

$$\bar{h}_H^T = Y^H N^H(-\bar{a}_y \psi^H(v, y) - \frac{\bar{a}_z i w}{u^2 + w^2} \frac{\partial \psi^H(v, y)}{\partial y}) \phi^c(w, z). \quad (1.17)$$

in which N^P are normalization coefficients (1.31), Z^P and Y^P are given by (1.32) and

$$\phi(w, z) = \exp(-i w z) \quad \text{and} \quad \phi^c(w, z) = (1/2\pi) \exp(i w z). \quad (1.18)$$

The scalar basis functions $\psi_i^P(v, y)$ are

$$\begin{aligned} R_{P0}^h \psi_0^P(v, y) \\ = \begin{cases} \exp(i v_0 y) + R_{P0}^h(v) \exp(-i v_0 y), & y \geq h \\ \exp[i v_1 (y-h)] \exp(i v_0 h) T_{P0}(v), & y \leq h \end{cases} \end{aligned} \quad (1.19)$$

$$\begin{aligned} R_{P1}^h \psi_1^P(v, y) \\ = \begin{cases} \exp[-i v_0 (y-h)] \exp(-i v_1 h) T_{P1}(v) & y \geq h \\ \exp(-i v_1 y) + R_{P1}^h(v) \exp(i v_1 y) & y \leq h \end{cases} \end{aligned} \quad (1.20)$$

and

$$\psi_s^P(v, y) = \psi_s^P(v, h) \begin{cases} \exp[-i v_{0s} (y-h)], & y \geq h \\ \exp[i v_{1s} (y-h)], & y \leq h, \end{cases} \quad (1.21)$$

in which the reflection coefficients with respect to the reference plane $y = 0$ (see Fig. 1.1) are

$$R_{P0}^h = R_{10}^P \exp(i 2 v_0 h) \quad R_{P1}^h = R_{01}^P \exp(-i 2 v_1 h) \quad (1.22)$$

where $R_{10}^P = -R_{01}^P$ are the Fresnel reflection coefficients

$$R_{10}^P(v) = R_{10}^P(C_0) = \frac{C_0 - C_1 K^P}{C_0 + C_1 K^P},$$

$$v_0 = k_0 C_0, \quad v_1 = k_1 C_1$$

$$\text{and } K^P = \begin{cases} \eta_1/\eta_0, & P=V \\ \eta_0/\eta_1, & P=H \end{cases}. \quad (1.23)$$

The intrinsic impedance for medium m is $\eta_m = (\mu_m/\epsilon_m)^{1/2}$. Furthermore in (1.19) and (1.20)

$$T_{P0} = 1 + R_{10}^P \text{ and } T_{P1} = 1 + R_{01}^P \quad (1.24)$$

are the transmission coefficients and $\psi_s^P(v, h)$ is the value of the surface wave basis function at the boundary $y = h$. For a homogeneous model of conducting ground the surface wave exists for the vertically polarized waves only. The superscripts or subscripts V and H are used to denote vertically or horizontally polarized waves. The symbol Σ_v denotes the summation (integration) over the entire wavenumber spectrum v consisting of the radiation term ($0 < v_0 < \infty$), the lateral wave term ($0 < v_1 < \infty$), and the guided surface wave term v_{is} . For $i = 0$ or 1

$$v_i = (k_i^2 - u^2 - w^2)^{1/2}, \quad \text{Im}(v_i) \leq 0 \quad (1.25)$$

and for the surface wave term v_{is} is a solution of the modal equation

$$-1/R_{10}^P = 1/R_{01}^P = 0 \quad P=V \text{ or } H \quad (1.26)$$

The wavenumber for medium $i = 0, 1$ is

$$k_i = \omega(\mu_i \epsilon_i)^{1/2}. \quad (1.27)$$

The basis functions satisfy the biorthogonal relationships for P and Q equal to V or H

$$\left. \begin{aligned} & \int_{-\infty}^{\infty} \bar{e}_T^P \cdot (\bar{h}_Q^T \times \bar{a}_x)' dydz \\ & \dots \\ & \int_{-\infty}^{\infty} \bar{h}_T^Q \cdot (\bar{a}_x \times \bar{e}_P^T)' dydz \end{aligned} \right\} = \delta_{P,Q} \Delta(v-v') \Delta(w-w') \quad (1.28)$$

in which, for the primed quantities the variables are u', v' , and w' and

$$\Delta(v-v') = \delta_{q,r} \times \begin{cases} \delta(v-v'), & v' \neq v_s \\ \delta_{v,v_s}, & v' = v_s \end{cases} \quad (1.29)$$

The subscripts q, r are equal to 0, 1 or s for the radiation, lateral wave and surface wave terms, respectively, and $\delta(\alpha-\beta)$ and $\delta_{\alpha,\beta}$ are the Dirac delta function and Kronecker delta, respectively. The completeness relationship is

$$\begin{aligned} \delta(y-y_0) &= \sum_v I^P N^P \psi^P(v, y) \psi^P(v, y_0) \\ &= \int_0^\infty I^P N_0^P \psi_0^P(v, y) \psi_0^P(v, y_0) dv_0 \\ &\quad + \int_0^\infty I^P N_1^P \psi_1^P(v, y) \psi_1^P(v, y_0) dv_1 \\ &\quad + I^P N_s^P \psi_s^P(v, y) \psi_s^P(v, y_0) \end{aligned} \quad (1.30)$$

The scalar basis functions for the radiation fields, and the lateral and surface waves are ψ_0^P , ψ_1^P , and ψ_s^P , respectively. The corresponding normalization coefficients are

$$N_s^P(v) = 1, \quad N_j^P(v) = R_{Pj}^h / 2\pi I_j^P(v), \quad j=0,1, \quad (1.31)$$

in which I_j^P are the transverse wave impedances or admittances. For $j=0,1$

$$I_j^P = \begin{cases} Z_j^V(v) = \frac{u^2 + w^2}{u\omega\epsilon_j} = \begin{cases} \frac{u^2 + w^2}{u\omega\epsilon_0} = Z_0^V(v) \\ \frac{u^2 + w^2}{u\omega\epsilon_1} = Z_1^V(v) \end{cases} \\ Y_j^H(v) = \frac{u^2 + w^2}{u\omega\mu_j} = \begin{cases} \frac{u^2 + w^2}{u\omega\mu_0} = Y_0^H(v) \\ \frac{u^2 + w^2}{u\omega\mu_1} = Y_1^H(v) \end{cases} \end{cases} \quad (1.32)$$

Thus in the above expressions for I_j^P (which are independent of y), the parameters for medium 0 are used for the radiation term, and the parameters for medium 1 are used for the lateral wave term. On the other hand $I^P(v, y)$ which is a function of y is given by

$$I^P(v, y) = \begin{cases} Z^V(v, y) = \begin{cases} Z_0^V, & y > h \\ Z_1^V, & y < h \end{cases} \\ Y^H(v, y) = \begin{cases} Y_0^H, & y > h \\ Y_1^H, & y < h \end{cases} \end{cases} \quad (1.33)$$

The irregular boundary and its gradient are assumed here to be continuous functions of x only. Thus the exact boundary conditions at $y = h(x)$ can be expressed exclusively in terms of the transverse field components

$$\left[\frac{1}{i\omega\epsilon} \nabla_T \cdot (\bar{H}_T \times \bar{a}_x) = -E_y \frac{dh}{dx} \right]_{h^-}^{h^+}, \left[E_z \right]_{h^-}^{h^+} = 0 \quad (1.34)$$

$$\left[\frac{1}{i\omega\mu} \nabla_T \cdot (\bar{a}_x \times \bar{E}_T) = -H_y \frac{dh}{dx} \right]_{h^-}^{h^+}, \left[H_z \right]_{h^-}^{h^+} = 0. \quad (1.35)$$

The complete field expansions are substituted into Maxwell's equations for the transverse field components and use is made of the orthogonality

relationship, Green's theorem, and the exact boundary conditions, to obtain the differential equations for the field transforms E^P and H^P . These may be expressed in terms of the forward and backward wave amplitudes a^P and b^P , respectively, as follows:

$$H^P = a^P + b^P \text{ and } E^P = a^P - b^P, \quad P = \begin{cases} V, & \text{upper sign} \\ H, & \text{lower sign} \end{cases} \quad (1.36)$$

Thus Maxwell's equations are converted into the following generalized telegraphists equations for $P=V$ or H (Bahar, 1973a,b)

$$-\frac{da^P}{dx} - iua^P = \sum_Q \sum_{v'} \int (S_{PQ}^{BA} a^Q + S_{PQ}^{BB} b^Q) dw' - A^P, \quad (1.37a)$$

and

$$-\frac{db^P}{dx} + iub^P = \sum_Q \sum_{v'} \int (S_{PQ}^{AA} a^Q + S_{PQ}^{AB} b^Q) dw' + B^P. \quad (1.37b)$$

The transmission scattering coefficients for the radiation fields are given by (Bahar, 1973b)

$$S_{PP}^{\alpha\beta}(v, v') = \frac{1}{2} \left[\frac{N_0^P(v)}{N_0^P(v')} C_P^P(v', v) - C_P^P(v, v') \right] I(w, w') \quad P=V, H, \alpha \neq \beta \quad (1.38a)$$

where

$$\frac{1}{2\pi} \int_{-\ell}^{\ell} \exp[i(w-w')z] dz \equiv I(w, w') \rightarrow \delta(w-w')$$

for $\ell \rightarrow \infty$ and

$$S_{HV}^{\alpha\beta}(v, v') = -S_{VH}^{\beta\alpha}(v', v) \frac{N_0^H(v)}{N_0^V(v')} = -\frac{1}{2} C_H^V(v, v') I(w, w') \quad (1.39)$$

The reflection scattering coefficients for the radiation fields are

$$S_{PP}^{\alpha\alpha}(v, v') = \frac{1}{2} \left[\frac{N_0^P(v)}{N_0^P(v')} C_P^P(v', v) + C_P^P(v, v') \right] I(w, w') \quad (1.40)$$

and

$$S_{HV}^{\alpha\alpha}(v, v') = S_{VH}^{\alpha\alpha}(v', v) \frac{N_0^H(v)}{N_0^V(v')} = \frac{1}{2} C_H^V(v, v') I(w, w') \quad (1.41)$$

The coupling coefficients C_P^P are given by

$$C_P^P(v', v) = \left[\frac{I^P(v, y) N^P(v')}{v'^2 - v^2} \left(\psi_0^P(v, y) \frac{\partial^2}{\partial x \partial y} \psi_0^P(v', y) - \frac{\partial}{\partial y} \psi_0^P(v, y) \frac{\partial}{\partial x} \psi_0^P(v', y) \right) \right]_{h^-}^{h^+} \quad (1.42)$$

On employing the properties of the scalar basis function ψ_0 , it can be shown that (1.42) reduces to

$$C_P^P(v', v) = \left[\frac{I^P(v, y) N_0^P(v')}{v'^2 - v^2} \left(v'^2 \psi_0^P(v, y) \psi_0^P(v', y) + \frac{\partial}{\partial y} \psi_0^P(v, y) \frac{\partial}{\partial y} \psi_0^P(v', y) \right) \right]_{h^+}^{h^-} \quad (1.43)$$

Similarly, it can be shown that

$$C_H^V(v', v) = \left\{ \frac{1wN_0^H(v')}{uu'} \left\{ \psi_0^V(v, y) \frac{\partial}{\partial x} \psi_0^H(v', y) + \frac{1}{k^2} \frac{\partial}{\partial y} \psi_0^V(v, y) \frac{\partial^2}{\partial x \partial y} \psi_0^H(v', y) + \frac{dh}{dx} \frac{(u^2 + w'^2)}{k^2} \frac{\partial}{\partial y} \psi_0^V(v, y) \psi_0^H(v', y) \right\} \right\}_{h^-}^{h^+} \quad (1.44)$$

Thus on using the properties of the scalar basis function ψ_0 ,

$$C_H^V(v', v) = \frac{1wN_0^H(v')}{uu'} \frac{dh}{dx} \cdot \left[\frac{\partial}{\partial y} \psi_0^V(v, y) \psi_0^H(v', y) - \psi_0^V(v, y) \frac{\partial}{\partial y} \psi_0^H(v', y) \right]_{h^-}^{h^+} - \frac{wN_0^H(v')}{uu'} \frac{dh}{dx} \psi_0^V(v, h) \psi_0^H(v', h) \cdot \left[v'_1 \left(1 - \frac{\mu_0}{\mu_1} \right) - v_1 \left(1 - \frac{\epsilon_0}{\epsilon_1} \right) \right] \quad (1.45)$$

Since excitations of vertically and horizontally polarized waves (with respect to the reference (x, z) plane) are considered, both vertical electric and magnetic dipoles located at $\vec{r} = \vec{r}^i = x^i \vec{a}_x + y^i \vec{a}_y + z^i \vec{a}_z = -r^i \vec{n}^i$ are treated in this analysis. Thus for vertical electric and magnetic dipoles of intensity J and M respectively the source terms A^P and B^P appearing in (1.37) are given by

$$\begin{aligned} A^V(v) &= -B^V(v) \\ &= -JZ_0^V(v)N_0^V(v)\psi_0^V(v, y^i) \\ &\quad \cdot \exp(i\omega z^i)\delta(x-x^i)/4\pi, \end{aligned} \quad (1.46)$$

$$\begin{aligned} A^H(v) &= B^H(v) \\ &= MY_0^H(v)N_0^H(v)\psi_0^H(v, y^i) \\ &\quad \cdot \exp(i\omega z^i)\delta(x-x^i)/4\pi \end{aligned} \quad (1.47)$$

The first-order iterative solutions for the wave amplitudes a^P and b^P are obtained by neglecting the transmission and reflection scattering coefficients in (1.37). These first-order solutions are substituted on the right side of (1.37), and the resulting equations are solved to obtain the second-order iterative solution for the wave amplitudes. These second-order iterative solutions are used in the complete expansions for the electromagnetic fields to obtain the desired iterative solutions for the scattered radiation fields through the use of the steepest descent method. Thus the first-order solutions to (1.37) are the unperturbed vertically and horizontally polarized fields excited by the vertical electric and magnetic dipoles respectively. The second-order iterative solutions which account for depolarization and scattering in arbitrary directions are suitable when multiple scattering can be ignored. Thus for $x > x^i$ the unperturbed wave amplitude excited by a vertical electric dipole of intensity J (amp-meter) is

$$\begin{aligned}
 a^V(x, v, w) &= H^V(x, v, w) \\
 &= \exp(-iux) \int_{-\infty}^x \exp(iux') A^V(x') dx' \\
 &= \exp[-iu(x-x_1)] A_0^V.
 \end{aligned} \tag{1.48}$$

where

$$A_0^V = \frac{-J}{4\pi} [N_0^V \psi_0^V \bar{\phi}]_{r=r}^{-1} \tag{1.49}$$

and

$$\bar{\phi}(w, z) = \exp(+iwz) \tag{1.50}$$

Substitute (1.48) into (1.8) to obtain the unperturbed magnetic field

$\bar{H}_T(x, y, z)$. Thus

$$H_z(x, y, z) = \int_{-\infty}^{\infty} \int_0^{\infty} H^V(x, v, w) \psi_0^V(v, y) \bar{\phi}(w, z) dv dw. \tag{1.51}$$

Noting that

$$[H^V/R_{V0}^h] = [H^V]_{-v_0} \tag{1.52}$$

it can be shown that for $y > h$ and $y^1 > h$, (1.51) can be written as

$$\begin{aligned}
 H_z(x, y, z) &= \frac{-J}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{V0}^h \psi_0^V \exp[-i(ux + wz)] \\
 &\quad \cdot \exp[i(ux^1 + wz^1 - v_0 y^1)] dv dw.
 \end{aligned} \tag{1.53}$$

When the vertical electric dipole is far from the rough surface ($kr^1 \gg 1$) the steepest descent method is used to evaluate (1.53) (Brekhovskikh, 1960).

$$\begin{aligned}
 u &= k_0 \sin \theta_0 \cos \phi, & v_0 &= k_0 \cos \theta_0, \\
 w &= k_0 \sin \theta_0 \sin \phi
 \end{aligned} \tag{1.54a}$$

$$\begin{aligned}
 x^1 &= -r^1 \sin \theta_0^1 \cos \phi^1, & y^1 &= r^1 \cos \theta_0^1, \\
 \text{and } z^1 &= -r^1 \sin \theta_0^1 \sin \phi^1
 \end{aligned} \tag{1.54b}$$

and

$$\begin{aligned}
 x &= r \sin \theta_0^f \cos \phi^f, & y &= r \cos \theta_0^f, \text{ and} \\
 z &= r \sin \theta_0^f \sin \phi^f.
 \end{aligned} \tag{1.54c}$$

It can be shown that the unperturbed magnetic field near the boundary $y = h$, is given by

$$H_z(x, y, z) \cong \frac{-iJ}{4\pi r^1} \exp(-ik_0 r^1) \cdot [u R_{V0}^h \psi_0^V \exp - i(ux + wz)]_{\Omega^1} \quad (1.55)$$

in which θ_0 and ϕ are replaced by θ_0^1 and ϕ^1 in the expressions for u , v , and w , and $\Omega^1(\theta_0^1, \phi^1)$ is the direction of the incident waves from the vertical dipole. Hence the azimuthal component of the incident unperturbed magnetic field at the origin is

$$H_\phi^1 = -H_z^1 / \cos \phi = ik_0 J \sin \theta_0^1 \exp(-ik_0 r^1) / 4\pi r^1. \quad (1.56)$$

To obtain the radiation field, transform use is made of (1.9). On integrating with respect to y and z and using the biorthogonal relationship (1.28) we obtain

$$\begin{aligned} H^V(x, v, w) &= H_z^1 R_{V0}^h(v^1) \exp(-iu^1 x) \\ &\quad \delta(w-w') \delta(v-v^1) \\ &\equiv a_0^1 \exp(-iu^1 x) \delta(w-w^1) \delta(v-v^1) = a^V(x, v, w) \end{aligned} \quad (1.57)$$

where $2l$ is the dimension of the surface in the z -direction ($-l < z < l$).

Thus for a vertical electric dipole at a large distance from the rough surface, the unperturbed field transform is represented by a Dirac delta function.

To obtain the second-order iterative solution that accounts for wave scattering, (1.57) is substituted on the right-hand side of the coupled differential equations for the wave amplitude $a^V(x, v, w)$. The resulting differential equation for the vertically polarized forward scattered wave amplitude is integrated to yield

$$a^V(x, v, w) = -\exp(-iux) \int_{-\infty}^x S_{VV}^{BA}(v, v^1) \cdot a_0^1 \exp i(u-u^1)x' dx' \quad (1.58)$$

A similar expression can be derived for the back scattered wave amplitude.

For an irregular boundary of dimension $2L$ in the x -direction, $y = h(x)$

$(-L < x < L)$, the forward scattered wave amplitude can be expressed as follows

$$a^V(x, v, w) = -M(\Omega, \Omega^1) \frac{\exp(-iux)}{u^2 - u^{12}} \cdot \int_{-L}^L \frac{dh}{dx} \exp i[(u-u^1)x + (v_0 + v_0^1)h] dx \quad (1.59)$$

Integrate (1.59) by parts to obtain the total (specular and non-specular) scattered wave amplitude (assuming that $k_0 L \gg 1$),

$$H^V(x, v, w) = a^V(x, v, w) = M(\Omega, \Omega^1) I(\Omega, \Omega^1, h, L) 2L \cdot \exp(-iux) / [(u+u^1)(v_0 + v_0^1)] \quad (1.60)$$

where $I(\Omega, \Omega^1, h, L)$ is given by

$$I(\Omega, \Omega^1, h, L) = \frac{1}{2L} \int_{-L}^L \exp\{ik_0[(C_0^1 + C_0)h + (S_0 \cos\phi - S_0^1 \cos\phi^1)x]\} dx \quad (1.61)$$

The forward scattered radiation field is obtained by substituting (1.60) into (1.51) using the relationships (1.52)

$$H_z(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H^V(x, v, w) \cdot \exp-i(v_0 y + wz) dv_0 dw \\ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{M(\Omega, \Omega^1) I(\Omega, \Omega^1, h, L) 2L}{(u+u^1)(v_0 + v_0^1)} \cdot \exp-i(ux + v_0 y + wz) dv_0 dw \quad (1.62)$$

Using (1.54) to express u, v_0 and w in terms of θ_0 and ϕ , and x, y , and z in terms of θ_0^f and ϕ^f the steepest descent method can be used to evaluate (1.62) for $k_0 r \gg 1$. Thus it can be shown that

$$H_z(x, y, z) = \frac{2\pi i u^f \exp(-ikr) M(\Omega^f, \Omega^i) I(\Omega^f, \Omega^i, h, L) 2L}{r(u^f + u^i)(v_0^f + v_0^i)} \quad (1.63)$$

The solutions for the other scattered fields are obtained in a similar manner. Thus the iterative solutions for the vertically and horizontally polarized scattered radiation fields $G^{Pf} = E^{Pf} = \eta_0 H^{Pf}$ can be expressed as follows

$$\begin{pmatrix} G^{Vf} \\ G^{Hf} \end{pmatrix} = G_0 I(\bar{n}^f, \bar{n}^i, \bar{r}_s, A) C_o^i \begin{pmatrix} F^{VV} & F^{VH} \\ F^{HV} & F^{HH} \end{pmatrix} \begin{pmatrix} G^{Vi} \\ G^{Hi} \end{pmatrix} \quad (1.64)$$

in which E^{Vf} , E^{Hf} and H^{Vf} , H^{Hf} are vertically and horizontally polarized components of the scattered electric and magnetic fields, respectively, and

$$G_0 = \frac{-ik_0^4 4L}{2\pi r^f} \exp[-ik_0 r^f] \quad , \quad C_o^i = \cos \theta_0^i \quad (1.65)$$

The projection of the rough surface in the (x, z) plane is $A_y = 4Ll$, and the observation point is at $\bar{r}^f = r^f \bar{n}^f$. The expression for the integral I in (1.64) is

$$I(\bar{n}^f, \bar{n}^i, \bar{r}_s, A) = \frac{1}{A_y} \int_{-l}^l \int_{-L}^L \exp[ik_0(\bar{n}^f - \bar{n}^i) \cdot \bar{r}_s] dx dz \quad (1.66)$$

in which \bar{n}^i and \bar{n}^f are unit vectors in the directions of the incident and the scattered wave normals in medium $y > h$, and \bar{r}_s is the distance from the origin to points on the rough surface.

For two dimensionally rough surfaces

$$\bar{r}_s = x\bar{a}_x + h(x, z)\bar{a}_y + z\bar{a}_z = \bar{r} - (y-h)\bar{a}_y = \bar{r} - f\bar{a}_y \quad (1.67a)$$

where

$$f(x,y,z) = y - h(x,z) = 0 \quad (1.67b)$$

is the equation for the rough surface, and the unit vector normal to the rough surface is

$$\begin{aligned} \bar{n} &= \nabla f / |\nabla f| \\ &\equiv \sin\gamma \cos\delta \bar{a}_x + \cos\gamma \bar{a}_y + \sin\gamma \sin\delta \bar{a}_z \\ &= (-h_x \bar{a}_x + \bar{a}_y - h_z \bar{a}_z) / (h_x^2 + 1 + h_z^2)^{1/2} \end{aligned} \quad (1.68a)$$

where

$$\frac{\partial h}{\partial x} = h_x \quad \text{and} \quad \frac{\partial h}{\partial z} = h_z \quad (1.68b)$$

In order to extend the solution (1.64) to problems of scattering by two-dimensionally rough surfaces $h(x,z)$ with arbitrary slopes, while preserving the relatively simple forms of these iterative solutions, the rough surface is regarded as a continuum of elementary surfaces of varying slope and height rather than a continuum of horizontal elementary surfaces of varying height. The contribution to the total scattered field from an inclined elementary surface is obtained from (1.64) after making the appropriate coordinate transformations. In this way the restrictions on the maximum slope of the rough surface are removed while preserving all the advantages of using the full-wave approach.

The contribution to the total scattered field from an elementary horizontal surfaces at $\bar{r} = \bar{r}_s$ of width $dx dz$ can be expressed in matrix form as

$$dG^f = G_0 dS G^i \quad (1.69)$$

in which the constant G_0 is defined in (1.65), G^i is the incident field at the origin, and

$$dS(x,z) = C_0^i F \exp[ik_0(\bar{n}^f - \bar{n}^i) \cdot \bar{r}_s] dx dz / A_y$$

$$\equiv C_0^i F dI(x,z) \quad (1.70)$$

The expression corresponding to $dS(x,z)$ for an elementary surface

$$d\bar{A} = \bar{n} dx dz / (\bar{n} \cdot \bar{a}_y) = \bar{n} dx dz / \cos \gamma \quad (1.71)$$

is given by dC . In matrix notation it is expressed as follows:

$$C = \int_{A_y} \int_{A_y} dC = \int_{A_y} \int_{A_y} C_0^{in} T^f F(\bar{n}^f, \bar{n}^i) T^i$$

$$\cdot \exp[ik_0(\bar{n}^f - \bar{n}^i) \cdot \bar{r}_s] d\bar{A} \cdot \bar{n} / A_y \quad (1.72)$$

In (1.70) the angles of incidence and scatter with respect to the reference plane (x,z) are replaced by the local angles of incidence and scatter with respect to the local tangent plane. The the elements of the matrix F (1.64) are (Bahar, 1981a,b)

$$C_0^{in} F^{VV} = \frac{2C_0^{in} C_0^{fn} [(\mu_r C_1^{in} C_1^{fn} \cos(\phi^{fn} - \phi^{in}) - S_0^{in} S_0^{fn})(1 - 1/\epsilon_r) + (1 - \mu_r) \cos(\phi^{fn} - \phi^{in})]}{(C_0^{in} + \eta_r C_1^{in})(C_0^{fn} + \eta_r C_1^{fn})(C_0^{in} + C_0^{fn})}$$

$$C_0^{in} F^{HH} = \frac{2C_0^{in} C_0^{fn} [(\epsilon_r C_1^{in} C_1^{fn} \cos(\phi^{fn} - \phi^{in}) - S_0^{in} S_0^{fn})(1 - 1/\mu_r) + (1 - \epsilon_r) \cos(\phi^{fn} - \phi^{in})]}{(C_0^{in} + C_1^{in}/\eta_r)(C_0^{fn} + C_1^{fn}/\eta_r)(C_0^{in} + C_0^{fn})}$$

$$C_0^{in} F^{HV} = \frac{-\sin(\phi^{fn} - \phi^{in}) 2C_0^{in} C_0^{fn} n_r [(1 - 1/\epsilon_r) C_1^{in} - (1 - 1/\mu_r) C_1^{fn}]}{(C_0^{in} + \eta_r C_1^{in})(C_0^{fn} + C_1^{fn}/\eta_r)(C_0^{in} + C_0^{fn})}$$

$$C_0^{in} F^{VH} = \frac{\sin(\phi^{fn} - \phi^{in}) 2C_0^{in} C_0^{fn} n_r [(1 - 1/\mu_r) C_1^{in} - (1 - 1/\epsilon_r) C_1^{fn}]}{(C_0^{in} + C_1^{in}/\eta_r)(C_0^{fn} + \eta_r C_1^{fn})(C_0^{in} + C_0^{fn})} \quad (1.73)$$

in which the dimensionless quantities n_r, η_r, ϵ_r , and μ_r are the refractive index, relative intrinsic impedance, relative permittivity, and relative permeability, respectively

$$\begin{aligned}
n_r &= (\epsilon_1 \mu_1 / \epsilon_0 \mu_0)^{1/2} \\
n_r &= n_1 / n_0 = \left(\frac{\mu_1}{\epsilon_1} / \frac{\mu_0}{\epsilon_0} \right)^{1/2} \\
\epsilon_r &= \epsilon_1 / \epsilon_0 \\
\mu_r &= \mu_1 / \mu_0
\end{aligned} \tag{1.74}$$

The cosines and sines of the angles of incidence and scatter (with respect to the local coordinate system) θ_0^{in} and θ_0^{fn} in free space, $y > h(x,z)$, are given by (see Fig. 1.2)

$$\begin{aligned}
C_0^{\text{in}} &= \cos \theta_0^{\text{in}} = \bar{n}^{\text{i}} \cdot \bar{n} \\
C_0^{\text{fn}} &= \cos \theta_0^{\text{fn}} = \bar{n}^{\text{f}} \cdot \bar{n}
\end{aligned} \tag{1.75a}$$

$$\begin{aligned}
S_0^{\text{in}} &= |\bar{n}^{\text{i}} \times \bar{n}| \\
S_0^{\text{fn}} &= |\bar{n}^{\text{f}} \times \bar{n}|
\end{aligned} \tag{1.75b}$$

The sines of the corresponding angles in medium 1, $y < h(x,z)$, are given by Snell's law

$$\begin{aligned}
S_1^{\text{in}} &= \sin \theta_1^{\text{in}} = S_0^{\text{in}} / n_r \\
S_1^{\text{fn}} &= \sin \theta_1^{\text{fn}} = S_0^{\text{fn}} / n_r
\end{aligned} \tag{1.76}$$

Thus

$$\begin{aligned}
C_1^{\text{in}} &= \cos \theta_1^{\text{in}} = [1 - (S_1^{\text{in}})^2]^{1/2} \\
C_1^{\text{fn}} &= \cos \theta_1^{\text{fn}} = [1 - (S_1^{\text{fn}})^2]^{1/2}
\end{aligned} \tag{1.77}$$

The cosine and sine of the angle between the planes of incidence and scatter in the local coordinate system are given by

$$\cos(\phi^{\text{fn}} - \phi^{\text{in}}) = \bar{a}_{\text{nf}}^{\text{n}} \cdot \bar{a}_{\text{Hi}}^{\text{n}} \tag{1.78a}$$

and

$$\sin(\phi^{\text{fn}} - \phi^{\text{in}}) = [\bar{a}_{\text{Hf}}^{\text{n}} \bar{a}_{\text{Hi}}^{\text{n}} \bar{n}] \tag{1.78b}$$

$$\begin{aligned}\bar{a}_{Hf}^f &= (\bar{n}^f \times \bar{a}_y) / |\bar{n}^f \times \bar{a}_y| \\ \bar{a}_{Hf}^n &= (\bar{n}^f \times \bar{n}) / |\bar{n}^f \times \bar{n}|\end{aligned}\quad (1.79)$$

and

$$\begin{aligned}\bar{a}_{Hi}^i &= (\bar{n}^i \times \bar{a}_y) / |\bar{n}^i \times \bar{a}_y| \\ \bar{a}_{Hi}^n &= (\bar{n}^i \times \bar{n}) / |\bar{n}^i \times \bar{n}|\end{aligned}\quad (1.80)$$

The vertically and horizontally polarized components of the incident and scattered electric and magnetic fields with respect to the local plane normal to the unit vector \bar{n} , are denoted by the subscript n. They are related to the components with respect to the reference plane through the following transformations

$$G^{in} \equiv \begin{pmatrix} E_n^{Vi} \\ E_n^{Hi} \end{pmatrix} = T^i G^i = \begin{pmatrix} C_\psi^i & S_\psi^i \\ -S_\psi^i & C_\psi^i \end{pmatrix} \begin{pmatrix} E^{Vi} \\ E^{Hi} \end{pmatrix} \quad (1.81)$$

and

$$G^f = \begin{pmatrix} E_n^{Vf} \\ E_n^{Hf} \end{pmatrix} = T^f G^{fn} = \begin{pmatrix} C_\psi^f & -S_\psi^f \\ S_\psi^f & C_\psi^f \end{pmatrix} \begin{pmatrix} E_n^{Vf} \\ E_n^{Hf} \end{pmatrix} \quad (1.82)$$

in which C_ψ^i and S_ψ^i are the cosine and sine of the angle between the local plane of incidence and reference plane of incidence normal to the unit vectors \bar{a}_{Hi}^n and \bar{a}_{Hi}^i , respectively. Thus they can be expressed in terms of the scalar product and the scalar triple product

$$\begin{aligned}C_\psi^i &= \cos\psi^i = \bar{a}_{Hi}^n \cdot \bar{a}_{Hi}^i \\ S_\psi^i &= \sin\psi^i = [\bar{a}_{Hi}^n \bar{a}_{Hi}^i \bar{n}^i]\end{aligned}\quad (1.83)$$

Similarly, C_ψ^f and S_ψ^f are the cosine and sine of the angle between the local plane of scatter and the reference plane of scatter normal to unit vectors \bar{a}_{Hf}^n and \bar{a}_{Hf}^f , respectively. Thus

$$\begin{aligned}
C_{\psi}^f &= \cos \psi^f = \bar{a}_{nf} \cdot \bar{a}_{nf}^n \\
S_{\psi}^f &= \sin \psi^f = [\bar{a}_{nf}^n \bar{a}_{nf}^f]
\end{aligned} \tag{1.84}$$

The full wave solutions for the scattered wave amplitudes can therefore be expressed as follows in matrix form

$$\begin{aligned}
G^f &= G_0 \int_A C_0^{in,f} F T^i \\
&\quad \cdot \exp[ik_0(\bar{n}^f - \bar{n}^i) \cdot \bar{r}_s] U(\bar{r}_s) d\bar{A} \cdot \bar{n} G^i \\
&\equiv G_0 C(\bar{n}^f, \bar{n}^i) G^i
\end{aligned} \tag{1.85}$$

In which the shadow function $U(\bar{r}_s)$ is

$$U(\bar{r}_s) = \begin{cases} 1 & \text{illuminated and visible region} \\ 0 & \text{nonilluminated or nonvisible region} \end{cases} \tag{1.86}$$

The normalized scattering cross section per unit area for rough surfaces is (Ishimaru, 1978)

$$\sigma^{PQ} = 4\pi (r^f)^2 |E^{Pf}|^2 / [A_y |E^{Q1}|^2] \tag{1.87}$$

in which A_y is the projection of the area of the rough surface A on the reference plane normal to \bar{a}_y . Thus for $P, Q = V, H$

$$\begin{aligned}
\sigma^{PQ} &= \frac{k_0^2}{\pi A_y} |C^{PQ}|^2 = \frac{k_0^2}{\pi A_y} \int D^{PQ}(\bar{r}_s) D^{PQ*}(\bar{r}_s') \\
&\quad \cdot \exp[ik_0(\bar{n}^f - \bar{n}^i) \cdot (\bar{r}_s - \bar{r}_s')] U(\bar{r}_s) U(\bar{r}_s') \frac{dx dy dx' dy'}{(\bar{n} \cdot \bar{a}_y)(\bar{n}' \cdot \bar{a}_y)}
\end{aligned} \tag{1.88}$$

where D^{PQ} is the element of the matrix

$$D = C_0^{in,f} F T^i \tag{1.89}$$

and the symbol $*$ denotes the complex conjugate.

For the incoherent, diffuse field the scattering cross section for unit cross-sectional area is given by (Ishimaru, 1978)

$$\langle \sigma^{PQ} \rangle = 4\pi (r^f)^2 \langle |E^{Pf} - \langle E^{Pf} \rangle|^2 \rangle / A_y |E^{Q1}|^2 \quad (1.90)$$

in which the symbol $\langle \rangle$ denotes the statistical average.

Thus the full wave expressions for the normalized scattering cross section per unit area is

$$\begin{aligned} \langle \sigma^{PQ} \rangle = \frac{k_0^2}{\pi A_y} & \left\langle \int_{A, A'} \left[S^{PQ} \exp\{i v_y (h-h')\} - \left[\frac{D^{PQU}}{\bar{n} \cdot \bar{a}_y} \exp(i v_y h) \right]^2 \right] \right. \\ & \cdot \exp[i v_x (x-x') + i v_z (z-z')] dx dz dx' dz' \end{aligned} \quad (1.91)$$

in which

$$\bar{v} = k_0 (\bar{n}^f - \bar{n}^i) = v_x \bar{a}_x + v_y \bar{a}_y + v_z \bar{a}_z \quad (1.92)$$

and

$$S^{PQ} = \frac{D^{PQ}(\bar{r}) U(\bar{r})}{\bar{n} \cdot \bar{a}_y} \frac{D^{PQ*}(\bar{r}') U(\bar{r}')}{\bar{n}' \cdot \bar{a}_y} \quad (1.93)$$

1.2 Interim Technical Reports and Publications in Scientific/Technical Journals

Preprints of the following manuscript "Scattering Cross Sections for Composite Models of Non-Gaussian Rough Surfaces for Which Decorrelation Implies Statistical Independence" were submitted to the contract monitor for publication as an Interim Technical Report (March 1, 1982-Nov. 30, 1982).

Abstract of Interim Report

The full wave approach is used to determine the scattering cross sections for composite models of non-Gaussian rough surfaces. It is assumed in this work that the rough surface heights become statistically independent when they decorrelate, thus no delta function type specular term appears in the expression for the scattered fields. The broad family of non-Gaussian surfaces considered range in the limit from exponential to Gaussian. It is seen that for small angles of incidence, the like polarized cross sections have the same dependence on the special form of the surface height joint probability density, but for large angles the scattering cross sections for the horizontally polarized waves are much more sensitive to the special form of the joint probability density. The corresponding results for the depolarized backscatter cross section are also presented. The shadow functions are shown to be rather insensitive to the special form of the joint probability density.

Preprints of the following manuscripts and conference papers were submitted to the Office of Public Affairs and project monitor for review and approval for publication:

1. "Propagation of Vertically and Horizontally Polarized Waves Excited by Distributions of Electric and Magnetic Sources in Irregular Stratified Spheroidal Structures of Finite Conductivity Generalized Field Transforms", Canadian Journal of Physics, Vol. 61 No. 1, pp. 113-127, 1983.
2. "Scattering and Depolarization of Electromagnetic Waves in Irregular Stratified Spheroidal Structures of Finite Conductivity--Full Wave Analysis", Canadian Journal of Physics, Vol. 61 No. 1, pp. 128-139, 1983.
3. "Scattering Cross Sections for Composite Surfaces That Cannot Be Treated as Perturbed-Physical Optics Problems," Radio Science, in press.
4. "Computations of Scattering Cross Sections for Composite Surfaces and the Specification of the Wavenumber Where Spectral Splitting Occurs", Submitted for review.
5. "Scattering and Depolarization by Rough Surfaces: Full Wave Approach", Proceedings of the SPIE International Technical Symposium of the International Society of Optical Engineering, Vol. 358 No. 28, pp. 1-14, August, 1982.
6. "Comparison of Backscatter Cross Sections for Composite Rough Surfaces with Different Mean Square Slopes", International Journal of Remote Sensing, in press.
7. "Shadowing by Non-Gaussian Rough Surfaces for Which Decorrelation Implies Statistical Independence", Radio Science, in press.
8. Joint International IEEE/APS and National Radio Science Meeting at the University of New Mexico, May 24-28, 1982, "Scattering Cross Sections for Composite Surfaces with Large Mean Square Slopes--Full Wave Analysis."
9. The SPIE 26th Annual International Technical Symposium of the International Society for Optical Engineering, San Diego, California, Aug. 23-27, 1982. Title of Invited Paper, "Scattering and Depolarization by Rough Surfaces."
10. International IEEE/APS Symposium and National Radio Science Meeting at the University of Houston, Texas, May 23-26, 1983, "Rough Surface Scattering that Cannot Be Analyzed Perturbed-Physical Optics Approaches."
11. International IEEE/APS Symposium and National Radio Science Meeting at the University of Houston, Texas, May 23-26, 1983, "Scattering Cross Sections for Composite Surfaces and the Wavenumber Where Spectral Splitting Occurs."

2.0 SCATTERING CROSS SECTIONS FOR COMPOSITE SURFACES THAT CANNOT BE TREATED AS PERTURBED-PHYSICAL OPTICS PROBLEMS

2.1 Background

Perturbation and physical optics theories have traditionally been used to derive the scattering cross sections for composite surfaces that can be regarded as small scale surface perturbations that ride on filtered, large scale surfaces. In this case perturbation theory accounts for Bragg scattering, while physical optics theory accounts for specular point scattering. However, for a more general class of composite surfaces that cannot be decomposed in such a manner, the perturbed-physical optics approach cannot be used. In these cases, it is shown, using the full wave approach, that the specular scattering associated with a filtered surface (consisting of the larger-scale spectral components), is strongly modified, and that Bragg scattering and specular point scattering begin to blend with each other. Since the full wave solution accounts for Bragg scattering as well as specular point scattering in a self-consistent manner, it is not necessary to filter (decompose) the composite surface to evaluate the scattering cross sections in the general case. However, filtering the composite surface enhances one's physical insight as to the validity (or lack thereof) of the perturbed-physical optics decomposition, and also facilitates the numerical evaluation of the scattering cross sections.

2.2 Discussion

In order to account for Bragg scattering as well as specular point scattering from random rough surfaces, composite models of the surface with different roughness scales have been considered (Wright 1968,

Barrick 1970, Barrick and Peake 1968). Thus, for instance, for backscatter more than 30° away from the vertical, it is shown through the use of perturbation theory (Rice 1951) that Bragg scattering is dominant and the scattering cross section, which is polarization dependent, is proportional to the surface height spectral density function. On the other hand, using physical optics theory (Beckmann 1968), the back scattering cross section near normal incidence is shown to be primarily due to specular point scattering and to be independent of polarization.

Using the composite models of Wright (1968), Semyonov (1966) and Valenzuela (1968) which are "mostly based on physical considerations," the rough surface is regarded as patches of slightly rough surfaces that ride over the large waves. Thus, in their work the scattering cross section associated with the surface with the small scale roughness is obtained by averaging over the distribution of slopes of the large scale roughness, or by averaging over the tilt angles in and perpendicular to the plane of incidence.

More recently Brown (1978, 1980) applied a combination of Burrows' perturbation theory (1967) and physical optics theory (Beckmann 1968) to derive the backscattering cross section from a perfectly conducting two scale model of rough surfaces. The first term in his solution is the specular point backscattering cross section associated with the large scale surface height h_l and the second term is the Bragg scattering cross section associated with the small scale surface height h_s . Thus, in his work it is necessary to decompose (i.e., spectrally filter) the composite surface. To this end, Brown's specification of the

wavenumber k_d (where spectral splitting is assumed to occur), is based entirely upon the characteristics of the small scale structure (Brown 1978). However, in the approaches of Hagfors (1966) and Tyler (1976) the specifications of k_d (spectral truncation) is assumed to be based on the characteristics of the large scale surface. In their approaches, however, they ignore the effects of the surface comprising the small scale spectral components ($k > k_d$) since their results are not meant to explain scattering far removed from the specular direction. In addition, it should be pointed out that even if the spectral components of the large scale (filtered) surface satisfy the radii of curvature criteria (associated with the Kirchhoff approximation of the surface fields), they may not necessarily satisfy the condition for deep phase modulation implicit in the evaluation of the specular point result (Barrick 1970).

In order to apply the full wave approach (Bahar 1981 a,b, 1982a,b) to problems of scattering by rough surfaces, it is not necessary to decompose (filter) the rough surface into one surface h_s with a small roughness scale, and another with large radii of curvature. However, when such a decomposition is feasible and the restrictions on both the large and small scale roughness are satisfied simultaneously, the full wave solutions for the scattering cross sections (which account for both Bragg scattering and specular point scattering in a self-consistent manner), can be expressed in terms of a weighted sum of two cross sections in agreement with perturbation and physical optics theories. A comparison between the full wave approach and the approaches of Brown and Valenzuela has also been made recently for surfaces with

moderately large mean square slopes (see Section 2.3).

The major objective of this work is to apply the full wave approach to composite surfaces which cannot be decomposed into a two scale model without violating either the small height variance criteria essential to the application of perturbation theory or the large radii of curvature criteria essential to the application of physical optics theory. Thus, in Section (2.3) the principal formulas for the scattering cross sections are developed for surfaces that do not satisfy either the small variance or the large radii of curvature restrictions. In Section (2.4) a composite surface which does not satisfy the physical optics (Kirchhoff) criteria is analyzed in detail. To decompose this surface it is assumed that the wavenumber k_d is specified in accordance with the criteria proposed by Tyler (1976) and that the surface that rides upon the filtered (Kirchhoff) surface may not satisfy the perturbation criteria. It is also assumed that if the wavenumber k_d were specified in accordance with the criteria based on the variance of the small scale surface (Brown 1978), the remaining large scale surface would not satisfy the standard physical optics criteria. Thus, a perturbed-physical optics approach cannot be applied to the problem. It is shown that the scattering cross sections for this general class of rough surfaces can be expressed as a weighted sum of two cross sections. The contribution associated with the filtered (Kirchhoff) surface is multiplied by a factor which is significantly smaller than unity and the term associated with the remaining surface that rides upon the filtered surfaces may contribute significantly even in the near specular direction. Thus for this general model it is illustrated that classic Bragg scattering and

specular point scattering begin to blend with each other. Since the full wave approach accounts for both Bragg and specular point scattering it is not necessary to decompose (filter) the composite surface when the full wave approach is applied to this broad class of scattering problems; however the decomposition may provide additional physical insight into prior approaches and facilitate the computation of the scattering cross sections.

2.3 Formulation of the Problem

Using the full wave solutions for the incoherent scattered radiation fields, the expression for the normalized scattering cross section per unit area (Ishimaru 1978) is given by

$$\langle \sigma^{PQ} \rangle = \frac{k_o^2}{\pi A_y} \left\langle \int_{A, A'} \left[S^{PQ} \exp\{i v_y (h-h')\} - \left| \frac{D^{PQU}}{\bar{n} \cdot \bar{a}_y} \exp(i v_y h) \right|^2 \right] \cdot \exp[i v_x (x-x') + i v_z (z-z')] dx dz dx' dz' \right\rangle \quad (2.1)$$

Second order iterative solutions for the scattered wave amplitudes are used in the derivation of (2.1) (Bahar 1981a,b; 1982a,b). This approximation of the full wave solution is suitable when multiple scattering can be neglected. In (2.1) A_y is the projection of the rough surface on the reference plane ($y=0$) (see Fig.2.1) and k_o is the free space wave number of the electromagnetic wave. The vector \bar{v} in the cartesian coordinate system (x, y, z) is

$$\bar{v} = \bar{k}^f - \bar{k}^i = k_o (\bar{n}^f - \bar{n}^i) = v_x \bar{a}_x + v_y \bar{a}_y + v_z \bar{a}_z \quad (2.2)$$

where \bar{n}^i and \bar{n}^f are unit vectors in the direction of the incident and scattered wave normals respectively. The coefficient S^{PQ} is (Bahar 1981a)

$$S^{PQ} = \frac{D^{PQ}(\bar{r}) U(\bar{r})}{\bar{n} \cdot \bar{a}_y} \frac{D^{PQ*}(\bar{r}') U(\bar{r}')}{\bar{n}' \cdot \bar{a}_y} \quad (2.3)$$

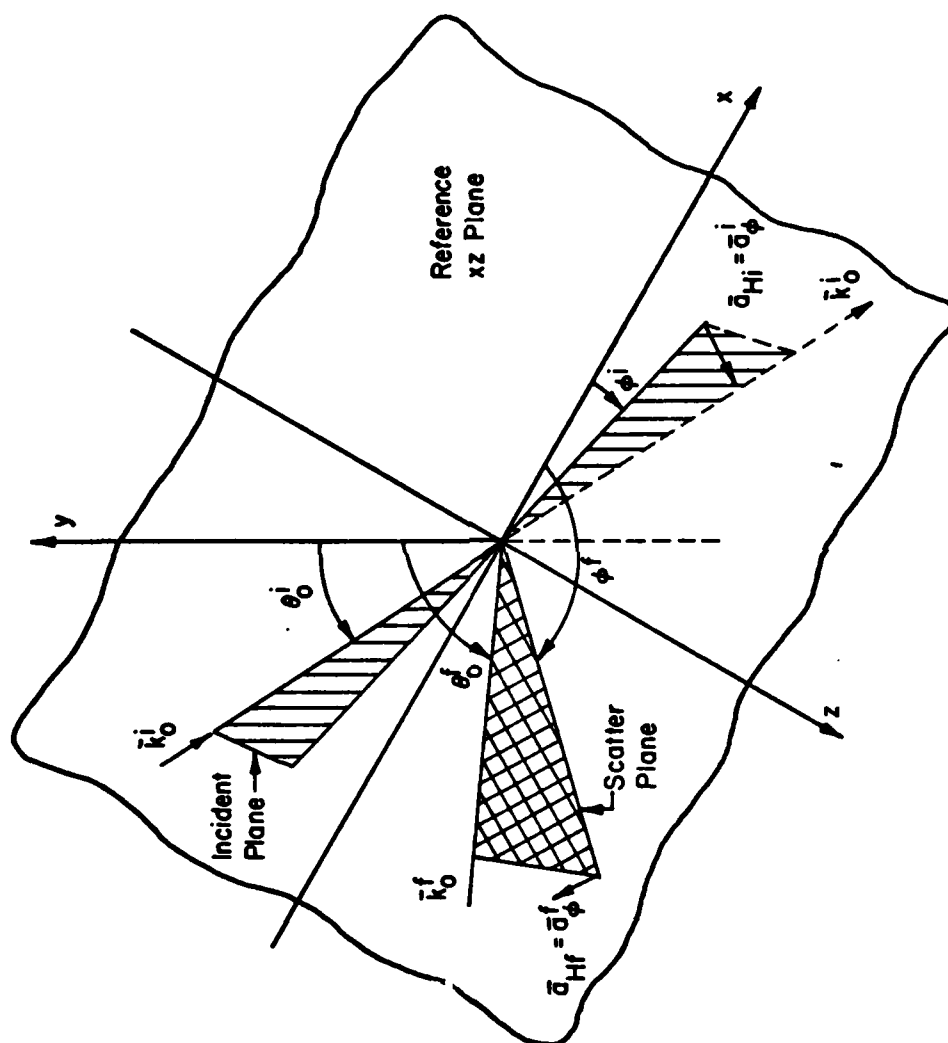


Fig. 2.1. Planes of incidence and scatter with respect to the reference coordinate system.

Mean (reference) plane for rough surface is $y = 0$.

in which D^{PQ} depends explicitly upon the polarization of the incident wave, (second superscript $Q=V$ - vertical, H - horizontal) and the polarization of the scattered wave (first superscript $P = V, H$), the directions of the incident and scattered wave normals \bar{n}^i and \bar{n}^f , the complex permittivity and permeability of the medium of propagation and the unit vectors, $\bar{n}(h_x, h_z)$, $\bar{n}'(h'_x, h'_z)$ normal to the rough surface at $\bar{r}(x, h, z)$ and $\bar{r}'(x', h', z')$. Thus for the random rough surface

$$f(x, y, z) = y - h(x, z) = 0 \quad (2.4a)$$

$$\nabla f = \nabla(y - h(x, z)) = (-h_x \bar{a}_x + \bar{a}_y - h_z \bar{a}_z) = \bar{n} |\nabla f| \quad (2.4b)$$

in which the components of the gradient of $h(x, z)$,

$$h_x = \partial h / \partial x, \quad h_z = \partial h / \partial z \quad (2.4c)$$

are random variables. The shadow function $U(\bar{r})$ is unity when the surface is both illuminated by the source and visible at the observation point and zero otherwise (Sancer 1969). When the surface height h and slopes (h_x, h_z) are statistically independent (a condition that holds for Gaussian surfaces at each point)

$$\left\langle \frac{D^{PQ} U}{\bar{n} \cdot \bar{a}_y} \exp(i v_y h) \right\rangle = \left\langle \frac{D^{PQ} U}{\bar{n} \cdot \bar{a}_y} \right\rangle \chi(v_y) \quad (2.5a)$$

in which $\chi(v_y)$ is the surface height characteristic function

$$\chi(v_y) = \langle \exp(i v_y h) \rangle = \int_{-\infty}^{\infty} \exp(i v_y h) p(h) dh \quad (2.5b)$$

In (2.5b) $p(h)$ is the surface height probability density function. The symbol $*$ denotes complex conjugate and the symbol $\langle \rangle$ denotes the statistical average.[†]

[†]As an example, for a Gaussian surface, $\chi(v_y) = \exp(-v_y^2 \langle h^2 \rangle / 2)$, where $\sigma^2 = \langle h^2 \rangle$ is the mean-square surface height.

If at high frequencies the higher order derivatives of the rough surface h can be neglected (Sancer 1969, Bahar 1981b; Bahar 1982a,b)

$$\langle S^{PQ} \exp\{iv_y(h-h')\} \rangle = \int \frac{D^{PQ}(\bar{r})U(\bar{r})D^{PQ*}(\bar{r}')U(\bar{r}')}{(\bar{n} \cdot \bar{a}_y)(\bar{n}' \cdot \bar{a}_y)} \exp\{iv_y[h_x(x-x') + h_z(z-z')]\} p(\bar{n}, \bar{n}', UU') d\bar{n} d\bar{n}' d(UU') \quad (2.6)$$

in which $p(\bar{n}, \bar{n}', UU')$ is the joint probability density function of the slopes $\bar{n}(h_x, h_z)$, $\bar{n}(h'_x, h'_z)$ and the shadow function product $U(\bar{r})U(\bar{r}')$ and

$$d\bar{n} \equiv dh_x dh_z, \quad d\bar{n}' \equiv dh'_x dh'_z \quad (2.7)$$

The joint probability density function $p(\bar{n}, \bar{n}', UU')$ can be expressed as follows:

$$p(\bar{n}, \bar{n}', UU') = p(\bar{n}, \bar{n}') p(UU' | \bar{n}, \bar{n}') \quad (2.8)$$

Assuming that h_x and h_z are independent variables (which is true for surfaces with isotropic roughness)

$$p(\bar{n}, \bar{n}') = p(h_x, h'_x) p(h_z, h'_z) \quad (2.9)$$

For a rough surface height with a Gaussian distribution, for example,

$$p(h_x, h'_x) = \frac{1}{2\pi\sigma_x^2 \sqrt{1-C_x^2}} \exp \left[-\frac{h_x^2 - 2C_x h_x h'_x + h'^2_x}{2\sigma_x^2(1-C_x^2)} \right] \quad (2.10a)$$

in which it is assumed that

$$\langle h_x \rangle = 0, \quad \langle h_x^2 \rangle = \sigma_x^2 \quad \text{and} \quad \langle h_x h'_x \rangle = \sigma_x^2 C_x \quad (2.10b)$$

For the homogeneous isotropic surface assumed in this work the normalized autocorrelation function C_x is a function of distance

$$\bar{r}_d = (x-x')\bar{a}_x + (z-z')\bar{a}_z = x_d\bar{a}_y + z_d\bar{a}_z \quad (2.11)$$

The density $p(h_z, h'_z)$ is given by (2.10) with h_x replaced by h_z and

$$\langle h_z \rangle = 0, \langle h_z^2 \rangle = \sigma_z^2 = \sigma_x^2, \langle h_z h_z' \rangle = \sigma_z^2 C_z = \sigma_x^2 C_x \quad (2.12)$$

The two point conditional density function $p(UU'|\bar{n},\bar{n}')$ is expressed in a form similar to the one given by Sancer (1969) for $p(U|\bar{n})$. Thus,

$$p(UU'|\bar{n},\bar{n}') = P_2(\bar{n}^f, \bar{n}^i | \bar{n}, \bar{n}') \delta(UU'-1) + [1-P_2(\bar{n}^f, \bar{n}^i | \bar{n}, \bar{n}')] \delta(UU'), \quad (2.13)$$

in which $\delta(\alpha)$ is the Dirac delta function and $P_2(\bar{n}^f, \bar{n}^i | \bar{n}, \bar{n}')$ is the probability that the points \bar{r} and \bar{r}' on the rough surface will be both illuminated by the source (incident wave normal, $\bar{n}^i(\theta^i, \phi^i)$) and visible at the observation point (scatter wave normal, $\bar{n}^f(\theta^f, \phi^f)$) given the value of the unit vectors normal to the surface \bar{n} and \bar{n}' , at these points. In (2.13)

$$P_2(\bar{n}^f, \bar{n}^i | \bar{n}, \bar{n}') = \begin{cases} P_2(\bar{n}^f, \bar{n}^i | \bar{n}) & , \text{ for } \bar{r} \rightarrow \bar{r}', C_x \rightarrow 1 \\ P_2(\bar{n}^f, \bar{n}^i | \bar{n}) P_2(\bar{n}^f, \bar{n}^i | \bar{n}'), & \text{ for } |\bar{r}-\bar{r}'| \rightarrow \infty, C_x \rightarrow 0 \end{cases} \quad (2.14)$$

and

$$P_2(\bar{n}^f, \bar{n}^i | \bar{n}) = P_2(\bar{n}^f, \bar{n}^i | \bar{n}_g) S(\bar{n}^f \cdot \bar{n}) S(-\bar{n}^i \cdot \bar{n}) \quad (2.15)$$

in which $P_2(\bar{n}^f, \bar{n}^i | \bar{n})$ is the probability that a point on the rough surface is both illuminated and visible given the value of the slopes at the point and $P_2(\bar{n}^f, \bar{n}^i | \bar{n}_g)$ is its value at the specular points $(\bar{n} \rightarrow \bar{n}_g)$ (Smith 1967, Sancer 1969). The arguments of the unit step functions $S(-\bar{n}^i \cdot \bar{n})$ and $S(\bar{n}^f \cdot \bar{n})$ vanish at points of the rough surface where the incident and scattered waves are tangent to the surface. Thus $S(-\bar{n}^i \cdot \bar{n}_g) = 1$ and $S(\bar{n}^f \cdot \bar{n}_g) = 1$ (Smith 1967, Brown 1980). In view of (2.14) and (2.15) it is assumed that

$$P_2(\bar{n}^f, \bar{n}^i | \bar{n}, \bar{n}') = \left[P_2(\bar{n}^f, \bar{n}^i | \bar{n}_s) \right]^{2 - (C_x^2 + C_z^2)/2} \cdot S(\bar{n}^f \cdot \bar{n}) S(\bar{n}^f \cdot \bar{n}') S(-\bar{n}^i \cdot \bar{n}) S(-\bar{n}^i \cdot \bar{n}') \quad (2.16)$$

Substitute (2.16) into (2.6) and integrate with respect to UU' to get

$$\langle S^{PQ} \exp[i v_y (h-h')] \rangle = \int \frac{D^{PQ}(\bar{r}) D^{PQ*}(\bar{r}')}{(\bar{n} \cdot \bar{a}_y) (\bar{n}' \cdot \bar{a}_y)} \exp[i v_y (h_x x_d + h_z z_d)] P_2(\bar{n}^f, \bar{n}^i | \bar{n}, \bar{n}') p(\bar{n}, \bar{n}') d\bar{n} d\bar{n}' \quad (2.17)$$

Since $\langle S^{PQ} \exp[i v_y (h-h')] \rangle$ is assumed to be a function of the distance $|\bar{r}_d|$, (2.12), the scattering cross section (2.1) reduces to

$$\langle \sigma^{PQ} \rangle = \frac{k_o^2}{\pi} \int_{-\infty}^{\infty} \left[\langle S^{PQ} \exp[i v_y (h-h')] \rangle - \left| \frac{D^{PQ} P_2(\bar{n}^f, \bar{n}^i | \bar{n})}{\bar{n} \cdot \bar{a}_y} \chi(v_y) \right|^2 \right] \cdot \exp[i v_x x_d + i v_z z_d] dx_d dz_d \quad (2.18)$$

In (2.18) the integrand vanishes when $|\bar{r}_d|$ is much larger than the correlation distances. The evaluation of the scattering cross section (2.18) simplifies considerably if it can be assumed that for $|\bar{r}_d|$ less than the correlation distances $\ell_x = \ell_z$ ($C_x(\ell_x) = C_z(\ell_z) = 1/e$ and e is the Neperian number) $\bar{n}(\bar{r}) \approx \bar{n}'(\bar{r}')$. In this case $p(\bar{n}, \bar{n}') \rightarrow p(\bar{n}) \delta(\bar{n} - \bar{n}')$ and

$$S^{PQ} \rightarrow \left| \frac{D^{PQ}(\bar{r}) U(\bar{r})}{\bar{n} \cdot \bar{a}_y} \right|^2 \quad (2.19)$$

Thus, for instance, when the radii of curvature of the rough surface are very large compared to the electromagnetic wavelength λ , or when the slope of the surface is small $\bar{n} \approx \bar{a}_y$

$$\langle S^{PQ} \exp[i v_y (h-h')] \rangle = \left\langle \int \left| \frac{D^{PQ}(\bar{r})}{\bar{n} \cdot \bar{a}_y} \right|^2 \exp[i v_y (h-h')] \cdot p(h_x, h_z) P_2(\bar{n}^f, \bar{n}^i | \bar{n}) dh_x dh_z \right\rangle \quad (2.20)$$

Equations (2.1) and (2.20) can be applied directly to composite surfaces (Brown 1978, 1980) given by

$$h = h_l + h_s \quad (2.21)$$

where h_l and h_s are assumed to be statistically independent random functions

$$k_o^2 \langle h_s^2 \rangle = k_o^2 \sigma_s^2 \ll 1, \quad \left| \frac{\partial h_s}{\partial x} \right| = |h_{sx}| \ll 1, \quad \left| \frac{\partial h_s}{\partial z} \right| = |h_{sz}| \ll 1, \quad (2.22a)$$

$$k_o^2 \langle h_l^2 \rangle = k_o^2 \sigma_l^2 \gg 1 \quad (2.22b)$$

and the radii of curvature of the surface h_l is much larger than the wavelength λ . In this case the full wave solution for the scattering cross section accounts for both specular point scattering as well as Bragg scattering, and (2.21) reduces to (Bahar 1981b, 1982a,b)

$$\langle \sigma^{PQ} \rangle = \langle \sigma^{PQ} \rangle_0 + \langle \sigma^{PQ} \rangle_1 \quad (2.23)$$

The first term in (2.23), $\langle \sigma^{PQ} \rangle_0$ is the specular scatter contribution

$$\begin{aligned} \langle \sigma^{PQ} \rangle_0 &= \frac{4\pi k_o^2}{v_y^2} \left[\left| \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right|^2 P_2(\bar{n}^f, \bar{n}^i | \bar{n}) p(n) |\chi^s(\bar{v} \cdot \bar{n})|^2 \right]_{\bar{n} \rightarrow \bar{n}_s} \\ &= |\chi^s(\bar{v} \cdot \bar{n}_s)|^2 \langle \sigma_\infty^{PQ} \rangle \end{aligned} \quad (2.24)$$

in which $\chi^s(\bar{v} \cdot \bar{n}_s) \approx 1$ is the characteristic function for the small scale surface height roughness, h_s , and $\langle \sigma_\infty^{PQ} \rangle$ is the physical optics scattering cross section (Bahar 1981a,b). The second term which accounts for Bragg scattering is given by

$$\langle \sigma^{PQ} \rangle_1 = \pi k_o^2 \int \frac{|D^{PQ}_{v_y^-}|^2}{\bar{n} \cdot \bar{a}_y} W(v_x^-, v_z^-) P_2(\bar{n}^f, \bar{n}^i | \bar{n}) p(h_x, h_z) dh_x dh_z \quad (2.25)$$

in which $W(v_x^-, v_z^-)$ is the small scale surface height spectral density (Barrick 1970, Ishimaru 1978).

$$W(\bar{v}_{\bar{x}}, \bar{v}_{\bar{x}}) = \frac{1}{\pi} \int_{-\infty}^{\infty} \langle \bar{h}_s(\bar{x}, \bar{z}) \bar{h}'_s(\bar{x}', \bar{z}') \rangle \exp(i\bar{v}_{\bar{x}} \bar{x}_d + i\bar{v}_{\bar{z}} \bar{z}_d) d\bar{x}_d d\bar{z}_d \quad (2.26)$$

In (2.26) it is assumed that the small scale surface autocorrelation function depends on distances $|\bar{x}_d \bar{n}_1 + \bar{z}_d \bar{n}_2|$ measured along the large scale surface $y-h_L(x,z) = 0$, and \bar{h}_s , the small scale surface height is measured penpendicular to the large scale surface. Furthermore

$$\bar{v} = \bar{v}_{\bar{x}} \bar{n}_1 + \bar{v}_{\bar{y}} \bar{n}_2 + \bar{v}_{\bar{z}} \bar{n}_3 \quad (2.27)$$

is the expansion of the vector \bar{v} in the local coordinate system (at any point on the large scale surface) associated with the unit vectors \bar{n}_1 , \bar{n}_2 , and \bar{n}_3 (Bahar 1982a,b) (see Fig. 2.2).

$$\bar{n}_1 = (\bar{n} \times \bar{a}_z) / |\bar{n} \times \bar{a}_z|, \quad \bar{n}_2 = \bar{n}, \quad \bar{n}_3 = \bar{n}_1 \times \bar{n} \quad (2.28)$$

The scattering cross section (2.25) is in complete agreement with perturbation theory (Rice 1951, Barrick 1970). Thus $\langle \sigma^{PQ} \rangle_1$ can be regarded as an average (over the distribution of slopes of the large scale surface) of the scattered power from patches of slightly rough surfaces that ride the large scale surface (Wright 1966, 1968, Valenzuela 1968). On expressing the unit vector \bar{n} in terms of the slope angles ψ and δ in and perpendicular to the plane of incidence, the scattering cross section (2.25), can be compared with earlier solutions that are "mostly based on physical considerations" (Valenzuela 1968, Valenzuela, Liang and Daley 1971, Bahar 1981c). The expression (2.25) is also in agreement with Brown's solution (that is expressed in terms of a two-dimensional convolution of transforms) provided that the mean squares of the large scale slopes $\sigma_x^2 = \sigma_z^2$ are very small. The difference between Brown's solution and the full wave solution arises because Brown (1978) (on

using a combination of Burrows' perturbation theory (1967) and physical optics (Beckmann 1968) assumes that the surface height autocorrelation function for the small scale surface ζ_s is dependent on distances in the reference (mean) plane rather than distances along the large scale surface, as assumed in this work and implicitly assumed by Valenzuela (1968). Furthermore, it should be pointed out that in Burrows' perturbation theory the small scale surface height is the distance from the unperturbed (filtered) surface to the perturbed surface measured along a line perpendicular to the unperturbed surface rather than perpendicular to the reference surface as assumed by Brown.

Since the full wave solution (2.23) accounts for both Bragg scattering as well as specular point scattering in a self-consistent manner without introducing a combination of perturbation and physical optics theories, it is not necessary to decompose the surface h_ℓ into large and small scale surface heights h_ℓ and h_s in order to evaluate the total scattering cross section $\langle \sigma^{PQ} \rangle$. Moreover, it is not necessary to restrict the application of the full wave approach only to surfaces that can be regarded as a small scale surface perturbation superimposed on a large scale surface roughness with very large radii of curvature compared to the wavelength. Thus in Section (2.4) the full wave approach is applied to more general composite rough surface models that may frequently fail to satisfy these restrictions.

2.4 Application to Surfaces That Do Not Satisfy Perturbation and Physical Optics Criteria

If the statistics of the rough surface are known for the entire

composite surface $h(x,z)$, the total scattering cross section can be evaluated using the full wave solution (2.18). Thus using the full wave approach it is not necessary to specify the wave number k_d at which spectral splitting between the two scale surface heights is assumed to occur. On the other hand, when different theories such as physical optics and perturbation theories are combined to analyze composite rough surfaces, it is necessary to spectrally decompose (filter) the rough surface. For instance, in Brown's work (1978) the specification of k_d is assumed to be based upon the characteristics of the small scale structure ($k_o^2 \langle h_s^2 \rangle \ll 1$). However, in the work by Tyler (1976), the specification of k_d is assumed to be based on the characteristics of the large scale surface. Thus, in order to apply physical optics theory to the large scale surface roughness, Tyler imposes the condition

$$P(|r_{12}^2| < \hat{r}_{12}^2) \ll 1 \quad (2.29)$$

in which $P(|r_{12}^2| < \hat{r}_{12}^2)$ is the probability that $|r_{12}^2|$ (the absolute value of the product of the principal radii of curvature of the large scale surface) is less than \hat{r}_{12}^2 , and the critical value for \hat{r}_{12}^2 is assumed to be k_o^{-2} (Tyler 1976). If both conditions (2.22b) ($k_o^2 \langle h_s^2 \rangle \ll 1$) as well as (2.29) ($P(|r_{12}^2| < \hat{r}_{12}^2) \ll 1$) are satisfied simultaneously, the scattering cross section is expressed as the sum of the specular point scattering cross section and the Bragg scattering cross section (2.23), since surfaces h_s and h_l individually satisfy the limitations imposed by perturbation and physical optical theories respectively. However, if both conditions are not satisfied simultaneously, the

physical optics model or the perturbation model or even a perturbed-physical optics model cannot be assumed in general.

To gain more physical insight and to facilitate the evaluation of the cross section (2.1), it is assumed here that the rough surface height h is decomposed into two surfaces h_F and \bar{h}_R (see Fig. 2.3). Thus

$$\bar{r} = \bar{r}_F(x, h_F, z) + \bar{h}_R \bar{n} \quad (2.30)$$

in which, following the filtering scheme proposed by Tyler (1976), the surface h_F satisfies the conditions $P(|r_{12}^2| < \hat{r}_{12}^2) \ll 1$ and $k_o^2 \langle h_F^2 \rangle \gg 1$. However, it is assumed that the remainder term \bar{h}_R does not satisfy the perturbation condition ($k_o^2 \langle \bar{h}_R^2 \rangle \ll 1$). Assuming that h_F and \bar{h}_R are statistically independent random functions

$$\langle \exp[i v_y (h-h')] \rangle \rightarrow \exp[i v_y (h_x^F x_d + h_z^F z_d)] \chi_2^R(v_y^-, -v_y^-) \quad (2.31a)$$

Thus the full wave solution (2.18) can be written as

$$\begin{aligned} \langle \sigma^{PQ} \rangle = \frac{k_o^2}{\pi} < \int_{-\infty}^{\infty} S^{PQ} \exp[i v_y (h_x^F x_d + h_z^F z_d)] (|\chi^R|^2 + \chi_2^R - |\chi^R|^2) \\ \exp[i v_x x_d + i v_z z_d] dx_d dz_d > \end{aligned} \quad (2.31b)$$

in which for convenience the term $|\chi^R|^2$ is added and subtracted and χ^R and χ_2^R are the characteristic and joint characteristic function for the surface height h_R respectively.

$$\chi_2^R(v_y^-, -v_y^-) = \langle \exp[i v_y (\bar{h}_R - \bar{h}_R')] \rangle \quad (2.31c)$$

In (2.31b) it is assumed that $|\chi^F|^2 \ll 1$ since $k_o^2 \langle h_F^2 \rangle \gg 1$. Thus, following the analytical procedures used in deriving (2.23) (Bahar 1981c), it can be shown that

$$\langle \sigma^{PQ} \rangle = \langle \sigma^{PQ} \rangle_F + \langle \sigma^{PQ} \rangle_R \quad (2.32a)$$

in which

$$\langle \sigma^{PQ} \rangle_F = \frac{k_o^2}{\pi} \left\langle \int_{-\infty}^{\infty} S^{PQ} \exp[iv_y(h_x^F x_d + h_z^F z_d)] |\chi^R|^2 \exp[iv_x x_d + iv_z z_d] dx_d dz_d \right. \\ \left. + |\chi^R(\bar{v} \cdot \bar{n}_s)|^2 \langle \sigma_{\infty}^{PQ} \rangle \right. \quad (2.32b)$$

and $\langle \sigma_{\infty}^{PQ} \rangle$ is the specular point scattering cross section for the filtered surface h_F . Since in this case it is assumed that $k_o^2 \langle \bar{h}_R^2 \rangle$ is not much smaller than unity, the factor in (2.32), $|\chi^R|^2$, can be significantly different from unity. On deriving the expressions for $\langle \sigma_{\infty}^{PQ} \rangle$ (2.24), and $\langle \sigma^{PQ} \rangle_F$ (2.32b), it is assumed that $v_y^2 \langle h_F^2 \rangle \gg 1$ and deep phase modulation occurs for all the roughness scales included in h_F .

Furthermore, the scattering cross section associated with the surface \bar{h}_R (that rides on the filtered surface) is

$$\langle \sigma^{PQ} \rangle_R = \frac{k_o^2}{\pi} \left\langle \frac{|D^{PQ}|^2 P_2(\bar{n}^f, \bar{n}^i | \bar{n})}{\bar{n} \cdot \bar{a}_y} [\chi_2^R(v_{\bar{y}}, -v_{\bar{y}}) - |\chi^R(v_{\bar{y}})|^2] \right. \\ \left. \cdot \exp[iv_{\bar{x}} \bar{x}_d + iv_{\bar{z}} \bar{z}_d] dx_d dz_d \right\rangle \quad (2.32c)$$

in which it is assumed that (2.19) is valid, $v_{\bar{x}}$, $v_{\bar{y}}$ and $v_{\bar{z}}$ are the components of \bar{v} in the local coordinate system associated with the filtered surface h_F (2.27), and $(\bar{x}_d^2 + \bar{z}_d^2)^{1/2}$ is distance measured along the filtered surface $y-h_F(x,z) = 0$ (see Fig. 2.3). It can be readily shown that if $k_o^2 \langle \bar{h}_R^2 \rangle \ll 1$ (2.32c) reduces to (2.25) with $\bar{h}_R = h_s$, since in this case

$$\chi_2^R(v_{\bar{y}}, -v_{\bar{y}}) - |\chi^R(v_{\bar{y}})|^2 \approx v_{\bar{y}}^2 \langle \bar{h}_R \bar{h}_R' \rangle \quad (2.33)$$

If the surface height \bar{h}_R is normally distributed and

$k_o^2 \langle \bar{h}_R^2 \rangle$ is not much smaller than unity

$$\chi_2^R(v_{\bar{y}}, -v_{\bar{y}}) - |\chi^R(v_{\bar{y}})|^2 = \exp(-v_{\bar{y}}^2 \langle \bar{h}_R^2 \rangle) [\exp(v_{\bar{y}}^2 \langle \bar{h}_R \bar{h}_R' \rangle) - 1]. \quad (2.34)$$

In this case (2.34) is expanded as follows

$$\chi_2^R(v_{\bar{y}}, -v_{\bar{y}}) - |\chi^R(v_{\bar{y}})|^2 = \exp(-v_{\bar{y}}^2 \langle \bar{h}_R^2 \rangle) \sum_{m=1}^{\infty} \frac{(v_{\bar{y}}^2 \langle \bar{h}_R \bar{h}_R' \rangle)^m}{m!} \quad (2.35)$$

and $\langle \sigma^{PQ} \rangle_R$ (3.22c) can be expressed as

$$\langle \sigma^{PQ} \rangle_R = \sum_{m=1}^{\infty} \langle \sigma^{PQ} \rangle_{Rm} \quad (2.36)$$

where

$$\begin{aligned} \langle \sigma^{PQ} \rangle_{Rm} &= \frac{k_o^2}{\pi} < \int \frac{|D^{PQ}|^2 P_2(\bar{n}^f, \bar{n}^i | \bar{n})}{\bar{n} \cdot \bar{a}_y} \exp(-v_{\bar{y}}^2 \langle \bar{h}_R^2 \rangle) \\ &\quad \cdot \frac{(v_{\bar{y}}^2 \langle \bar{h}_R \bar{h}_R' \rangle)^m}{m!} \exp(i v_{\bar{x}} \bar{x}_d + i v_{\bar{z}} \bar{z}_d) d\bar{x}_d d\bar{z}_d > \\ &= 4\pi k_o^2 < \frac{|D^{PQ}|^2 P_2(\bar{n}^f, \bar{n}^i | \bar{n})}{\bar{n} \cdot \bar{a}_y} \exp(-v_{\bar{y}}^2 \langle \bar{h}_R^2 \rangle) \cdot \left(\frac{v_{\bar{y}}}{2} \right)^{2m} \frac{W_m(c_{\bar{x}}^-, v_{\bar{z}}^-)}{m!} > \end{aligned} \quad (2.37)$$

and

$$\begin{aligned} \frac{W_m(v_{\bar{x}}, v_{\bar{z}})}{2^{2m}} &= \frac{1}{(2\pi)^2} \int (\langle \bar{h}_R \bar{h}_R' \rangle)^m \exp(i v_{\bar{x}} \bar{x}_d + i v_{\bar{z}} \bar{z}_d) d\bar{x}_d d\bar{z}_d \\ &= \frac{1}{2^{2m}} \int W_{m-1}(v_{\bar{x}}', v_{\bar{z}}') W_1(v_{\bar{x}} - v_{\bar{x}}', v_{\bar{z}} - v_{\bar{z}}') dv_{\bar{x}}' dv_{\bar{z}}' \\ &\equiv \frac{1}{2^{2m}} W_{m-1}(v_{\bar{x}}, v_{\bar{z}}) \otimes W_1(v_{\bar{x}}, v_{\bar{z}}) \end{aligned} \quad (2.38)$$

In (2.38) the symbol \otimes denotes the two dimensional convolution of W_{m-1} with W . Since $\frac{W_1(v_{\bar{x}}, v_{\bar{z}})}{4} = \frac{W(v_{\bar{x}}, v_{\bar{z}})}{4}$ is the two dimensional Fourier

transform of the surface height autocorrelation function $\langle \bar{h}_R \bar{h}_R' \rangle$, the

term $\langle \sigma^{PQ} \rangle_{R1}$ accounts for first-order Bragg scattering. However, since $4k_o^2 \langle \bar{h}_R^2 \rangle$ is not much smaller than unity the factor $\exp(-v_y^2 \bar{h}_R^2)$ appearing in $\langle \sigma^{PQ} \rangle_{R1}$ is significantly smaller than unity for backscatter near normal incidence and approaches unity for backscatter near grazing incidence. This factor does not appear in the solution derived on the basis of perturbation theory (2.25).

When the autocorrelation function $\langle \bar{h}_R \bar{h}'_R \rangle$ can be expressed analytically in closed form, the two dimensional Fourier transforms in (2.38) can be integrated directly. For example, when the Gaussian form is used,

$$\langle \bar{h}_R \bar{h}'_R \rangle = \exp((-x_d^2 - z_d^2)/T^2) \quad (2.39)$$

closed form expression can be derived for $W_m(v_{\bar{x}}, v_{\bar{z}})$ which are also Gaussian (Beckmann and Spizzichino, 1963). However, since in practical problems the surface height \bar{h}_R is usually characterized explicitly by its spectral density function $W(v_{\bar{x}}, v_{\bar{z}})$, and since this function is rarely Gaussian for natural surfaces, the Fourier transforms in (2.38) are evaluated through the repeated use of the two dimensional convolution theorem.

If the mean square slope of the surface height is very small compared to unity ($\bar{n} \rightarrow \bar{a}_y$)

$$\begin{aligned} \langle \sigma^{PP} \rangle_{R2} &\approx 4\pi k_o^2 |D^{PQ}|_{\bar{n}=\bar{a}_y}^2 P_2(\bar{n}^f, \bar{n}^i | \bar{a}_y) \exp(-v_y^2 \langle \bar{h}_R^2 \rangle) \\ &\quad \left(\frac{v_y}{2} \right)^4 \frac{W_2(v_{\bar{x}}, v_{\bar{z}})}{2} \end{aligned} \quad (2.40)$$

The expression $\langle \sigma^{PQ} \rangle_{R2}$ can be readily evaluated for backscatter near normal incidence. Thus assuming a perfectly conducting surface, for $\vec{n}^f = -\vec{n}^i = \vec{a}_y$

$$\langle \sigma^{PQ} \rangle_{R2} \rightarrow 2\pi k_o^6 \exp(-4k_o^2 \langle \bar{h}_R^2 \rangle) W_2(0,0) \quad (2.41)$$

Assuming, for example, as isotropic surface height spectral density of the form

$$W(k) = \begin{cases} A/k^4, & k_1 < k < k_2 \\ 0, & k < k_1 \text{ and } k > k_2 \end{cases} \quad (2.42)$$

$$W_2(0,0) = \int_0^{2\pi} \int_{k_1}^{k_2} W^2(k) k dk d\phi = \frac{\pi A^2}{3} \left[\frac{1}{k_1^3} - \frac{1}{k_2^3} \right] \quad (2.43)$$

The mean square of the surface height \bar{h}_R is

$$\langle \bar{h}_R^2 \rangle = \int_0^{2\pi} \int_{k_1}^{k_2} \frac{W(k)}{4} k dk d\phi = \frac{\pi A}{4} \left[\frac{1}{k_1^2} - \frac{1}{k_2^2} \right] \quad (2.44)$$

Thus for $k_2^2 \gg k_1^2$

$$W_2(0,0) = \left(\frac{\pi A^2}{3} \frac{4 \langle \bar{h}_R^2 \rangle^3}{\pi A} \right) \quad (2.45)$$

and (2.41) reduces to

$$\langle \sigma^{PP} \rangle_{R2} \rightarrow \frac{2}{3\pi A} (4k_o^2 \langle \bar{h}_R^2 \rangle)^3 \exp(-4k_o^2 \langle \bar{h}_R^2 \rangle) \quad (2.46)$$

As in (2.25), the contribution $\langle \sigma^{PQ} \rangle_R$, (2.32c) to the scattering cross section is due to scattering by the surface \bar{h}_R that rides on the filtered surface h_F . In general however, if $k_o^2 \langle \bar{h}_R^2 \rangle$ is not much smaller than unity

the contribution to the scattering cross section $\langle \sigma^{PQ} \rangle_R$, (2.32c) cannot be derived on the basis of perturbation theory and the factor $|\chi^R|^2$ in (2.32b) cannot be replaced by unity. Thus a combination of perturbation theory and physical optics theory cannot be applied to this problem. Physically, the coefficient $|\chi^R|^2$ in (2.32b) accounts for the fact that when $k_o^2 \langle \bar{h}_R^2 \rangle$ is not much smaller than unity, the surface irregularities \bar{h}_R that ride on the filtered surface h_F could significantly reduce the contribution of the "specular" scattering cross section, $\langle \sigma^{PQ} \rangle_F$, associated with the filtered surface h_F . For backscatter, the perturbation term associated with Bragg scatter $\langle \sigma^{PQ} \rangle_1$ (2.25) is much smaller than the term associated with specular scatter $\langle \sigma^{PQ} \rangle_o$ (2.24) near normal incidence and $\langle \sigma^{PP} \rangle_1 \gg \langle \sigma^{PP} \rangle_o$ for near grazing incidence provided that (2.22) and the radius of curvature criteria are satisfied. However, for the more general case treated in this section a similar relationship between $\langle \sigma^{PP} \rangle_F$ (2.32b) and $\langle \sigma^{PP} \rangle_R$ may not exist. Thus, while it is certainly not necessary to decompose (filter) the rough surface height h into surfaces h_F and \bar{h}_R (2.30) in order to derive the scattering cross sections when the full wave approach is used, such a decomposition does provide additional physical insight. The results of the analysis carried out in this section illustrate how the specular scattering and Bragg scattering components of the total scattering cross section begin to blend with each other as the value of $k_o^2 \langle \bar{h}_R^2 \rangle$ increases and the composite surface can no longer be regarded as a perturbed-physical optics (Kirchhoff) surface. Furthermore, the decomposition (filtering) of the surface height assists in making the result (2.1) easier to compute.

It can also be shown that if instead of (2.30) the criteria for decomposing the composite surface is based only on the characteristics of the small scale surface height h_s , (2.22b) such that

$$h = h_s + h_R \quad (2.47)$$

and if the radii of curvature associated with the remaining surface h_R are not large compared to wavelength (such that the physical optics theory cannot be applied to it) the full wave approach can still be used.

2.5 Concluding Remarks

It is shown that since the full wave theory accounts for both Bragg scattering as well as specular point scattering in a self-consistent manner, in order to evaluate the scattering cross sections it is not necessary to use a combination of perturbation and physical optics theories. Furthermore, it is shown that the general formulas derived in Section (2.3) can be applied to surfaces that do not necessarily satisfy the perturbation theory restrictions $k_o^2 \langle \bar{h}_R^2 \rangle \ll 1$ and/or the physical optics theory restrictions on the radii of curvature (2.29). For the general case considered in Section (2.4), in which a perturbed-physical optics approach cannot be used, it is illustrated how Bragg scattering and specular point scattering begin to blend with each other. In this case, decomposition (filtering) not only enhances one's physical insight, but also facilitates the numerical evaluation of the scattering cross sections (2.1). These numerical evaluations need to be pursued with special emphasis on the specification of k_d (where spectral splitting is assumed to occur).

3.0 COMPUTATIONS OF SCATTERING CROSS SECTIONS FOR COMPOSITE SURFACES AND THE SPECIFICATION OF THE WAVENUMBER WHERE SPECTRAL SPLITTING OCCURS

3.1 Background

The scattering cross sections for composite random rough surfaces are evaluated using the full wave approach. They are compared with earlier solutions based on a combination of perturbation theory which accounts for Bragg scattering and physical optics which accounts for specular point theory. The full wave solutions which account for both Bragg scattering and specular point scattering in a self-consistent manner are expressed as a weighted sum of two cross sections. The first is associated with a filtered surface, consisting of the larger scale spectral components, and the second is associated with the surface consisting of the smaller scale spectral components. The specification of the surface wavenumber that separates the surface with the larger spectral components from the surface with the smaller spectral components is dealt with in detail. Since the full wave approach is not restricted by the limitations of perturbation theory, it is possible to examine the sensitivity of the computed values for the backscatter cross sections to large variations in the value of the wavenumber where spectral splitting is assumed to occur.

3.2 Discussion

In this section, the backscatter cross sections for composite models of rough surfaces are evaluated using the full wave solutions to the problem (Bahar and Barrick 1982). The like-polarized backscatter cross sections for both vertically and horizontally polarized waves as

well as the cross-polarized backscatter cross sections are evaluated as functions of the angle of incidence. In order to compare the full wave results with earlier solutions for the backscatter cross sections, the full wave solutions are expressed in terms of a weighted sum of scattering cross sections. The first is associated with a filtered surface h_F consisting of the larger scale spectral components ($k < k_d$). The second is associated with a surface h_R consisting of the smaller scale spectral components ($k > k_d$) (see Section 2).

In an attempt to draw more definite conclusions regarding the choice of k_d (the wavenumber where spectral splitting is assumed to occur) between the surfaces h_F and h_R , the wavenumber k_d is varied over a wide range of values. The wavenumber k_d is related to the parameter $\beta = 4k_o^2 \langle h_R^2 \rangle$, where k_o is the wavenumber for the electromagnetic wave and $\langle h_R^2 \rangle$ is the mean square of the surface height h_R . Thus, on applying a perturbed-physical optics approach to rough surface scattering (Brown 1978, 1980), the wavenumber k_d was chosen on the basis of the characteristics of the surface consisting of the small scale spectral components. However, in the approaches of Hagfors (1966) and Tyler (1976), the specification of the wavenumber k_d is assumed to be based on the characteristics of the filtered surface h_F . In their approaches, however, they ignore the effects of the surface h_R (consisting of the smaller spectral components, $k > k_d$) since their results were meant to explain backscatter from lunar and planetary surfaces at near normal incidence.

It should be pointed out that even if the spectral components of the filtered surface density satisfy the radii of curvature criteria (associated with the Kirchhoff approximations of the surface fields),

they may not necessarily satisfy the condition for deep phase modulation implicit in the evaluation of the specular point result (Barrick 1970).

In Section (3.3) of this report the principal full wave expressions for the normalized scattering cross sections are summarized. Since the parameter β need not be restricted by considerations of perturbation theory, it is assumed that the filtered surface h_F satisfied the radii of curvature criteria (associated with the Kirchhoff approximations for the surface fields) as well as the conditions for deep phase modulation.

In Section (3.4) an extensive set of numerical data is presented for the backscatter cross sections associated with the like-polarized and the cross-polarized waves. It is shown that while, as expected, the cross sections associated with the individual surfaces h_F and h_R critically depend on the choice of β (and therefore k_d), the total backscatter cross section remains practically insensitive to β for $\beta \geq 1.0$. However, there are small, though perhaps significant, differences in the values for the total cross sections where β is increased from 0.1 to 1.0. It should be pointed out that these differences (as β varies from 0.1 to $\beta = 1.0$) are significantly smaller than those predicted on the basis of Brown's analysis. The reason why the full wave results for the total backscatter cross sections merge for values of $\beta \geq 1.0$ is related to the condition for deep phase modulation. Some of the key observations of Brown, in his contribution to rough surface scattering, (1978), are summarized in Section (3.4) to emphasize the problems related to the specification of k_d . However, the interested researcher in this field should familiarize himself with his work as well as the pioneering

contributions of Hagfors (1966) and Tyler (1976).

The details given in the illustrative examples of Section (3.4) are presented primarily for tutorial purposes and to vividly establish the criteria for specifying k_d . Thus, the engineer need not consider the evaluation of the cross section for $\beta > 1.0$ or concern himself with the individual terms that add up to determine the total cross sections. Nevertheless it does enhance the engineer's physical insight in dealing with problems of rough surface scattering. For example, the full wave approach can explain why the frequency dependence of the backscatter cross sections changes as one changes the angle of incidence or when one varies the frequency of the electromagnetic wave.

3.3 Formulation of the Problem

In this section the principal expressions for the normalized scattering cross sections per unit area are summarized and the full wave solutions are compared with earlier solutions based on perturbation and physical optics theories.

The expressions for the normalized scattering cross sections per unit area (Ishimaru 1978), based on the full wave solutions for the incoherent scattered radiation fields, are given by (Bahar 1981a)

$$\begin{aligned} \langle \sigma^{PQ} \rangle = \frac{k_o^2}{\pi} \int_{-\infty}^{\infty} \left[\langle S^{PQ} \exp[i v_y (h-h')] \rangle - \left| \frac{D^{PQ} p_2(\bar{n}^f, \bar{n}^i | \bar{n})}{\bar{n} \cdot \bar{a}_y} \right| \chi(n_y) \right] \\ \cdot \exp[i v_x x_d + i v_z z_d] dx_d dz_d \end{aligned} \quad (3.1)$$

in which

$$\bar{r}_d = (x-x')\bar{a}_x + (z-z')\bar{a}_z = x_d\bar{a}_x + z_d\bar{a}_z \quad (3.2)$$

is the radius vector between two points in the reference plane (x, z)

(see Fig. 3.1). The vector \bar{v} in the reference coordinate system (x,y,z) is

$$\bar{v} = k_0 (\bar{n}^f - \bar{n}^i) = v_x \bar{a}_x + v_y \bar{a}_y + v_z \bar{a}_z \quad (3.3)$$

where k_0 is the free space wavenumber for the electromagnetic wave and \bar{n}^i and \bar{n}^f are unit vectors in the direction of the incident and scattered wave normals respectively. An $\exp(i\omega t)$ time dependence is assumed throughout this work. The symbol $\langle \rangle$ denotes the statistical average and it can be shown that (Bahar 1981b)

$$\begin{aligned} \langle S^{PQ} \exp[i v_y (h-h')] \rangle = & \left\langle \left| \frac{D^{PQ}(\bar{r})}{\bar{n} \cdot \bar{a}_y} \right|^2 \exp[i v_y (h-h')] \right. \\ & \left. P(h_x, h_z) P_2(\bar{n}^f, \bar{n}^i | \bar{n}) dh_x dh_z \right\rangle \end{aligned} \quad (3.4)$$

where \bar{n} is the unit vector normal to the rough surface (see Fig. 3.1).

$$f(x,y,z) = y - h(x,z) = 0 \quad (3.5a)$$

Thus

$$\nabla f = \bar{n} |\nabla f| = \nabla (y - h(x,z)) = (-h_x \bar{a}_x + \bar{a}_y - h_z \bar{a}_z) \quad (3.5b)$$

in which the components of the gradient of $h(x,z)$

$$h_x = \partial h / \partial x, \quad h_z = \partial h / \partial z \quad (3.5c)$$

are random variables and $p(h_x, h_z)$ is the distribution function for the slopes h_x and h_z . The expression for $\langle \sigma^{PQ} \rangle$ (3.1) accounts for shadowing and

$$P_2(\bar{n}^f, \bar{n}^i | \bar{n}) = P_2(\bar{n}^f, \bar{n}^i | \bar{n}_s) S(\bar{n}^f \cdot \bar{n}) S(-\bar{n}^i \cdot \bar{n}) \quad (3.6)$$

in which $P_2(\bar{n}^f, \bar{n}^i | \bar{n})$ is the probability that a point in the rough surface is both illuminated and visible given the value of the slopes at the point (Smith 1967; Sancer 1969) and $P_2(\bar{n}^f, \bar{n}^i | \bar{n}_s)$ is its value at the specular

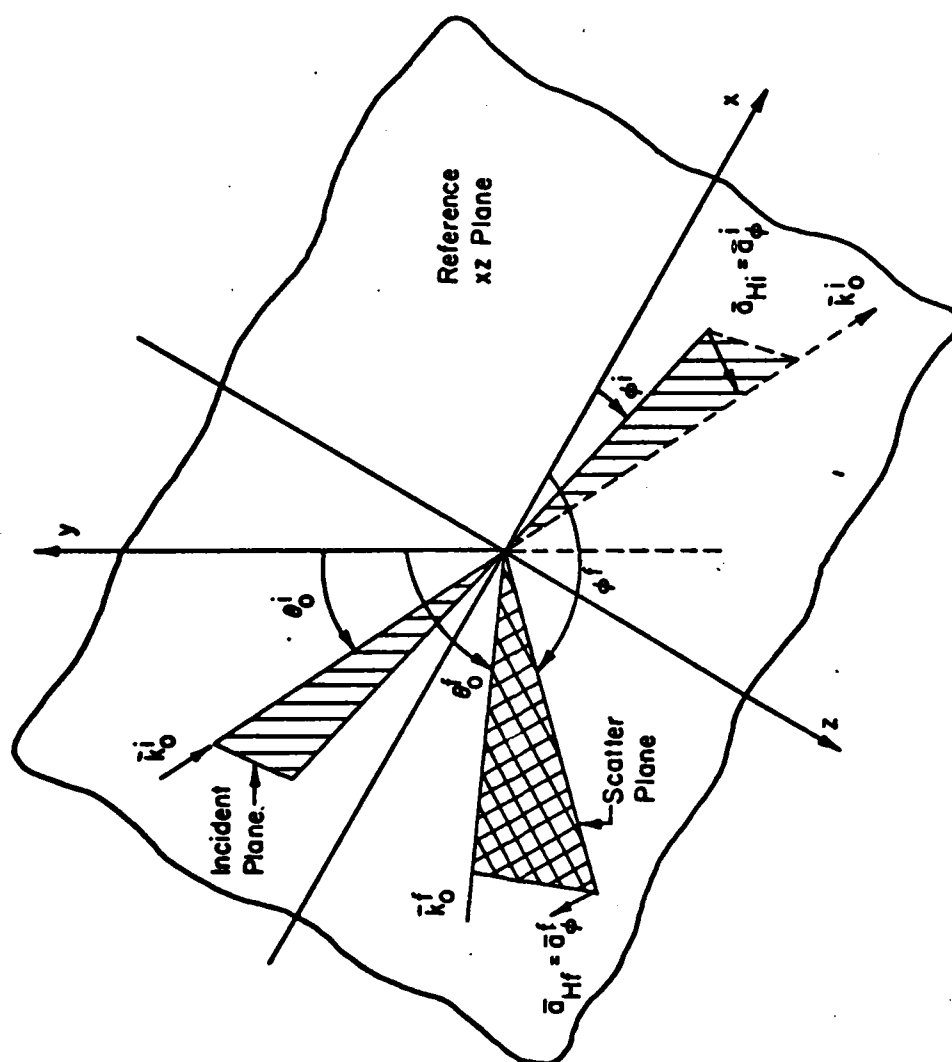


Fig. 3.1. Planes of incidence and scatter with respect to the reference coordinate system.

Mean (reference) plane for rough surface is $y = 0$.

points where the unit vector \bar{n} is given by

$$\bar{n} \rightarrow \bar{n}_s = \frac{\bar{n}^f - \bar{n}^i}{|\bar{n}^f - \bar{n}^i|} = \bar{v}/v \quad (3.7)$$

The arguments of the unit step functions $S(-\bar{n}^i \cdot \bar{n})$ and $S(\bar{n}^f \cdot \bar{n})$ vanish at points of the rough surface where the incident and scattered waves are tangent to the surface. Thus $S(-\bar{n}^i \cdot \bar{n}_s) = 1$ and $S(\bar{n}^f \cdot \bar{n}_s) = 1$. The characteristic function and the joint characteristic function for the surface height h are respectively,

$$\chi(v_y) = \langle \exp(iv_y h) \rangle \quad (3.8)$$

and

$$\chi_2(v_y, -v_y) = \langle \exp iv_y (h-h') \rangle \quad (3.9)$$

For rough surface heights with Gaussian distributions (assumed in this work)

$$\chi(v_y) = \exp(-v_y^2 \langle h^2 \rangle / 2) \quad (3.10)$$

in which $\langle h^2 \rangle = \sigma^2$ is the mean square surface height. Furthermore,

$$\chi_2(v_y) = \exp[-v_y^2 (\langle h^2 \rangle - \langle h h' \rangle)] \quad (3.11)$$

where $\langle h(x,z)h'(x',z') \rangle$ is the surface height autocorrelation function. The coefficients D^{PQ} (Bahar 1981a) depend explicitly upon the polarization of the incident wave (second superscript $Q=V$ - vertical, $Q=H$ - horizontal) and the polarization of the scattered wave (first superscript $(P=V,H)$), the direction of the incident and scattered wave normals \bar{n}^i and \bar{n}^f respectively, the complex permittivity and permeability of the medium of propagation ϵ and μ respectively and the unit vector \bar{n} normal to the rough surface. In the above expression for $\langle \sigma^{PQ} \rangle$, (3.1), it has been assumed that the surface height h and slopes (h_x, h_z) are statistically independent

a condition that holds for Gaussian surfaces at each point) and that for distances $|\bar{r}_d|$ less than the correlation distances ℓ (where the surface height autocorrelation function $C(\ell) = C(0)/e$)

$$\bar{n}(h_x, h_z) \approx \bar{n}'(h'_x, h'_z) \quad (3.12)$$

If the statistics for the entire rough surface $h(x, z)$ are known, the total scattering cross sections can be evaluated using the full wave solution (3.1). However, when a combination of perturbation theory (Rice 1951; Barrick 1970) and physical optics theory (Beckmann 1968) is applied to the problem of rough surface scattering, a two scale model of the surface is used and the surface is decomposed into a filtered surface height (consisting of the large scale spectral components of the surface height) and a small scale surface height h_s that is superimposed (rides on) the large scale filtered surfaces (Wright 1968, Valenzuela 1968; h_s consists of the smaller scale spectral components of the surface height). To this end it is necessary to specify the wavenumber k_d at which spectral splitting is assumed to occur. For instance, Brown (1978) who uses a combination of Burrows' perturbation theory (1967) and physical optics (Beckmann 1968), bases the specification of k_d upon the characteristics of the small scale structure ($k_0^2 \langle h_s^2 \rangle \ll 1$). However, in the works by Hagfors (1966) and Tyler (1976), the specification of k_d is assumed to be based upon the characteristics of the large scale (filtered) surface h_F . Thus, in order to justify the application of physical optics theory to the large scale surface roughness, Tyler (1976) imposes the condition

$$P(|r_{12}^2| < \hat{r}_{12}^2) \ll 1 \quad (3.13)$$

in which $P(|r_{12}^2| < \hat{r}_{12}^2)$ is the probability that $|r_{12}^2|$ (the absolute value

of the product of the principal radii of curvature of the large scale surface) is less than \hat{r}_{12}^2 , and the critical value for \hat{r}_{12}^2 is assumed to be k_0^{-2} (Tyler 1976).

Since the full wave approach accounts for both specular point scattering as well as Bragg scattering in a self-consistent manner, it is not necessary to filter (decompose) the rough surface to evaluate the scattering cross sections. However, filtering the composite surface enhances one's physical insight as to the validity (or lack thereof), of the perturbed-physical optics approach to the scattering problem and also facilitates the numerical evaluation of the cross section. Thus it is assumed here that the rough surface height $h(x,z)$ is decomposed into two surfaces such that the position vector to a point on the rough surface is

$$\bar{r}_s = \bar{r}_F(x, h_F, z) + \bar{n} \bar{h}_R \quad (3.14)$$

In order to apply the physical optics-specular scattering approximation to the filtered surface h_F (consisting of the larger scale spectral components) assume that h_F satisfies the appropriate radii of curvature criteria (for example (3.13)). In addition, assume that deep phase modulation occurs. Thus, the distances from the transmitter and receiver to the individual specular points (which are random variables) are such that the contributions from the individual specular points are distributed uniformly in phase from $-\pi$ to π . One should also note that no matter how the composite surface is filtered, the physical optics approach (based on the Kirchhoff approximations for the surface fields) is not valid if for a given \bar{n}^i and \bar{n}^f specular points do not exist on the rough surface (Bahar 1981a,b). Let \bar{h}_R consist of the remaining part of the rough surface spectrum ($k > k_d$). Since in this case k_d

is specified by the desired characteristics of the filtered surface h_F , it is not assumed here that $k_o^2 \langle \bar{h}_R^2 \rangle$ is much smaller than unity. Thus perturbation theory (Rice 1951; Barrick 1970), cannot be applied to the remaining part of the surface height \bar{h}_R . For truly random, rough natural surfaces such as the sea surface, it is assumed that h_F and \bar{h}_R are statistically random functions. If in addition, it is assumed that $\nabla h \approx \nabla h_F$, the full wave solution (3.1) can be expressed as a weighted sum of the individual cross sections for the surfaces h_F and \bar{h}_R respectively (Bahar 1981b).

$$\langle \sigma^{PQ} \rangle = \langle \sigma^{PQ} \rangle_F + \langle \sigma^{PQ} \rangle_R \quad (3.15)$$

The first term $\langle \sigma^{PQ} \rangle_F$ can be shown to be given by

$$\langle \sigma^{PQ} \rangle_F = |\chi^R(\bar{v} \cdot \bar{n}_s)|^2 \langle \sigma_\infty^{PQ} \rangle, \quad (3.16)$$

in which $\chi^R(\bar{v} \cdot \bar{n}_s) = \chi^R(v)$ is the characteristic function for the surface \bar{h}_R and $\langle \sigma_\infty^{PQ} \rangle$ is the specular point scattering cross section for the filtered surface h_F . The factor $\chi^R(v)$ that multiplies $\langle \sigma_\infty^{PQ} \rangle$ accounts for the degradation of the specular points contributions due to the superimposed surface \bar{h}_R (Bahar 1981a). It can be shown that

$$\langle \sigma_\infty^{PQ} \rangle = \frac{4\pi k_o^2}{v_y^2} \left[\left| \frac{D^{PQ}}{\bar{n} \cdot \bar{a}_y} \right|^2 P_2(\bar{n}^f, \bar{n}^i | \bar{n}) P(\bar{n}) \right]_{\bar{n} \rightarrow \bar{n}_s} \quad (3.17)$$

The second term $\langle \sigma^{PQ} \rangle_R$ is the scattering cross section for the surface that rides on the filtered surface h_F . It can be expressed as follows (Bahar and Barrick 1982)

$$\langle \sigma^{PQ} \rangle_R = \sum_{m=1}^{\infty} \langle \sigma^{PQ} \rangle_{R_m} \quad (3.18)$$

where

$$\langle \sigma^{PQ} \rangle_{R_m} = 4\pi k_o^2 \int \frac{|D^{PQ}|^2 P_2(\bar{n}^f, \bar{n}^i | \bar{n})}{\bar{n} \cdot \bar{a}_y} \exp(-v_y^2 \langle \bar{h}_R^2 \rangle) \left(\frac{v_y}{2} \right)^{2m} \frac{W_m(v_x^-, v_z^-)}{m!} p(h_x, h_z) dh_x dh_z \quad (3.19)$$

in which v_x^- , v_y^- and v_z^- are the components of \bar{v} (3.3) in the local coordinate system (at each point on the large scale surface) associated with the unit vectors \bar{n}_1 , \bar{n}_2 , and \bar{n}_3 (see Fig. 3.2). Thus \bar{v} in (3.3) is also expressed as (Bahar 1982)

$$\bar{v} = v_x^- \bar{n}_1 + v_y^- \bar{n}_2 + v_z^- \bar{n}_3 \quad (3.20)$$

where

$$\bar{n}_1 = (\bar{n} \times \bar{a}_z) / |\bar{n} \times \bar{a}_z|, \quad \bar{n}_2 = \bar{n}, \quad \bar{n}_3 = \bar{n}_1 \times \bar{n} \quad (3.21)$$

The function $W_m(v_x^-, v_z^-) / 2^{2m}$ is the two dimensional Fourier transform of $(\langle \bar{h}_R \bar{h}_R' \rangle)^m$

$$\begin{aligned} \frac{W_m(v_x^-, v_z^-)}{2^{2m}} &= \frac{1}{(2\pi)^2} \int (\langle \bar{h}_R \bar{h}_R' \rangle)^m \exp(i v_x^- \bar{x}_d + i v_z^- \bar{z}_d) d\bar{x}_d d\bar{z}_d \\ &= \frac{1}{2^{2m}} \int W_{m-1}(v_x^-, v_z^-) W_1(v_x^- - v_x^-, v_z^- - v_z^-) dv_x^- dv_z^- \\ &= \frac{1}{2^{2m}} W_{m-1}(v_x^-, v_z^-) \otimes W_1(v_x^-, v_z^-) \end{aligned} \quad (3.22)$$

In (3.22) $|\bar{x}_d \bar{a}_x + \bar{z}_d \bar{a}_z|$ is the distance measured along the large scale surface and the symbol \otimes denotes the two dimensional convolution of W_{m-1} with W_1 . Since $W_1(v_x^-, v_z^-) / 4 = W(v_x^-, v_z^-) / 4$, is the two dimensional Fourier transform of the surface height autocorrelation function $\langle \bar{h}_R \bar{h}_R' \rangle$, it is equal to the spectral density for the surface height \bar{h}_R . Thus, the first term in (3.18), $\langle \sigma^{PQ} \rangle_{R1}$, accounts for first order Bragg scattering.

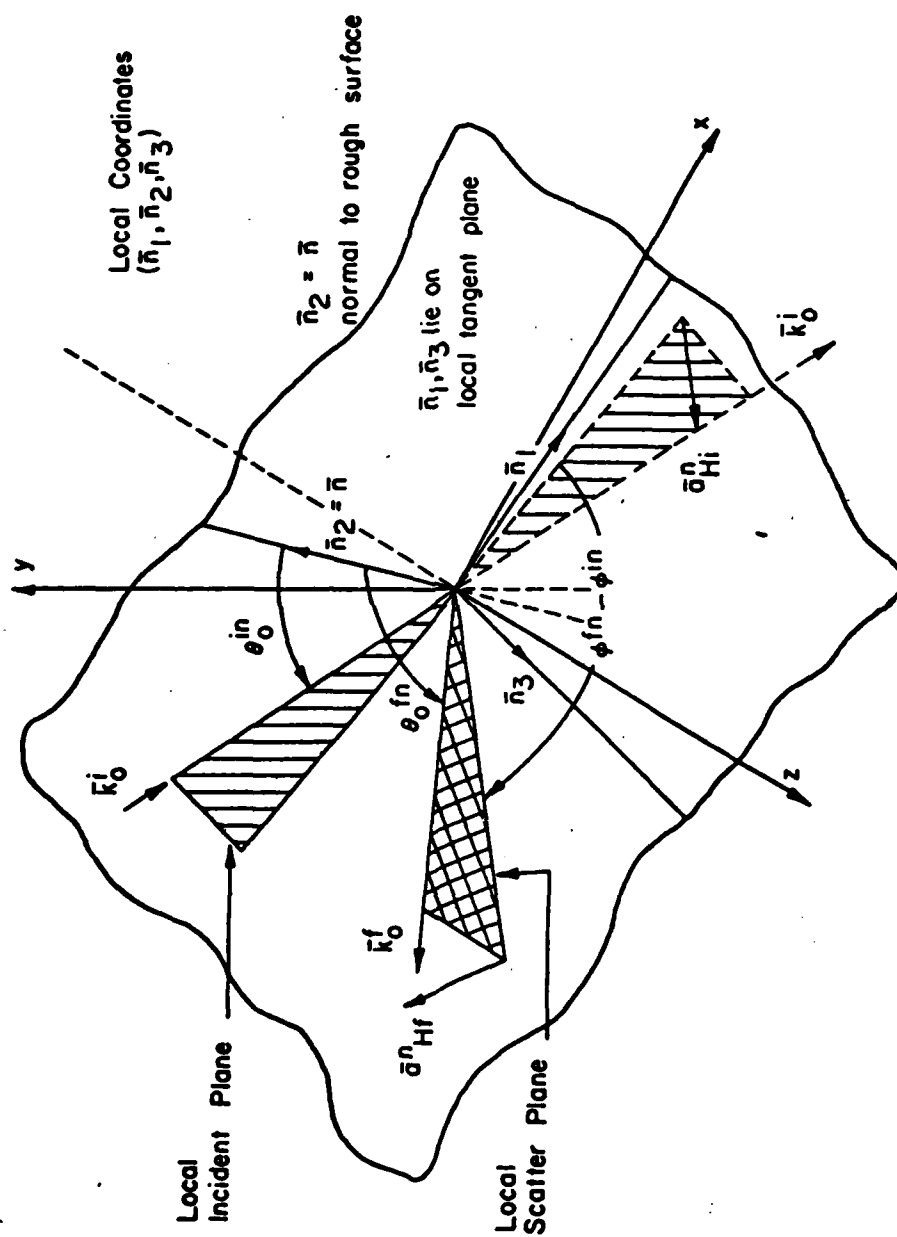


Fig. 3.2. Local planes of incidence and scatter and local coordinate system with unit vectors $\vec{n}_1, \vec{n}_2, \vec{n}_3$.

However, when $4k_o^2 \langle \bar{h}_R^2 \rangle$ is not much smaller than unity the factor $\exp(-v_y^2 \langle \bar{h}_R^2 \rangle)$ appearing in the expression for $\langle \sigma^{PQ} \rangle_{Rl}$ is significantly smaller than unity for backscatter near normal incidence and approaches unity for backscatter near grazing incidence. This factor as well as the coefficient $\chi^R(v)$ in the expression for $\langle \sigma^{PQ} \rangle_F$ (3.16) do not appear in the expressions for the scattering cross sections based on a perturbed-physical optics solution. Moreover, the arguments of $W(v_x, v_z)$ are the components of \bar{v} in the local tangent plane and not in the reference plane (x, z) . Thus for $k_o^2 \langle \bar{h}_R^2 \rangle \ll 1$, $\langle \sigma^{PQ} \rangle_R \approx \langle \sigma^{PQ} \rangle_{Rl}$ can be regarded as an average (over the distribution of slopes of the large scale surface) of the scattered power from patches of slightly rough surfaces that ride the large scale surface (Wright 1966, 1968; Valenzuela 1968). The scattering cross section $\langle \sigma^{PQ} \rangle_{Rl}$ can be compared with earlier solutions that are "mostly based on physical considerations" on expressing the unit vector \bar{n} in terms of the slope angles ψ and δ in and perpendicular to the plane of incidence (Valenzuela 1968; Valenzuela, Laing and Daley 1971; Bahar 1982b). For $k_o^2 \langle \bar{h}_R^2 \rangle \ll 1$ the expression for $\langle \sigma^{PQ} \rangle_{Rl}$ is also in agreement with Brown (1978) (as corrected in Brown 1980), provided that the mean squares of the large scale slopes are small. This difference arises primarily because Brown assumes that the surface height autocorrelation function for the small scale surface is dependent on distances $|\bar{r}_d| = (x_d^2 + z_d^2)^{1/2}$ (3.2) in the reference (mean) plane rather than distances $(\bar{x}_d^2 + \bar{z}_d^2)^{1/2}$ measured along the large scale surface as assumed in this work and implicitly by Valenzuela (1968). Moreover, using the perturbation

method (Burrows 1967), the small scale surface height \bar{h}_R is measured normal to the filtered surface h_F rather than normal to the reference plane ($y=0$) as assumed by Brown (1978).

As the parameter $\beta = 4k_0^2 \langle \bar{h}_R^2 \rangle$ increases and the corresponding value for k_d (the wavenumber at which spectral splitting is assumed to occur) decreases, it is necessary to retain an increasing number of terms in (3.18). However, since the full wave approach accounts for specular point scattering as well as Bragg scattering in a self-consistent manner, the numerical value for the total scattering cross section $\langle \sigma^{PQ} \rangle$ (3.15) should not depend on the specific value of k_d , provided that the filtered surface h_F satisfies not only the radii of curvature criteria but also the condition for deep phase modulation assumed in reducing (3.1) to the form (3.15). This property of the full wave solution is demonstrated in the next section where illustrative examples are presented.

3.4 Illustrative Examples

In order to compare the full wave solutions for the scattering cross sections (3.15) with earlier solutions appearing in the technical literature (Brown 1978, 1980), the following specific form for the total surface height spectrum is selected

$$W_T(v_x^-, v_z^-) = \left(\frac{2}{\pi}\right) S(v_x^-, v_z^-) = \begin{cases} \left(\frac{2}{\pi}\right) B k^4 / (k^2 + k_c^2)^4 & k \leq k_c \\ 0 & k > k_c \end{cases} \quad (3.23)$$

where W is the spectral notation originally used by Rice (1953) and S is the notation used by Brown (1978). For the assumed isotropic model of the ocean surface

$$B = 0.0046 \quad (3.24a)$$

$$k^2 = v_x^2 + v_z^2, (\text{cm})^{-2}, k_c = 12 (\text{cm})^{-1} \quad (3.24b)$$

$$\kappa = (335.2 V^4)^{-1/2} (\text{cm}^{-1}), V = 4.3 (\text{m/s}) \quad (3.24c)$$

in which V , the surface wind speed is given in m/s. The wavelength for the electromagnetic wave is

$$\lambda_0 = 2(\text{cm}), (k_0 = 3.1416 (\text{cm})^{-1}) \quad (3.25)$$

The mean square height for the surface h_R is given by

$$\langle h_R^2 \rangle = \int_0^{2\pi} \int_{k_d}^{k_c} \frac{W_T(k)}{4} k dk d\phi \approx \frac{B}{2} \left[\frac{1}{k_d^2} - \frac{1}{k_c^2} \right] \quad (3.26)$$

and the mean square slope for the filtered surface h_F is

$$\sigma_{FS}^2 = \langle h_{FS}^2 \rangle = \int_0^{2\pi} \int_0^{k_d} \frac{W_T(k)}{4} k^3 dk d\phi \approx \frac{B}{2} \left[\frac{-11}{6} + \ln \left(\frac{k_d^2 + \kappa^2}{\kappa^2} \right) \right] \quad (3.27)$$

In (3.26) and (3.27) it is assumed that $k_d \gg \kappa$. The slope distribution function is assumed to be Gaussian, thus

$$P(h_x, h_z) = \frac{1}{\pi \sigma_{FS}^2} \exp \left(-\frac{h_x^2 + h_z^2}{\sigma_{FS}^2} \right) \quad (3.28)$$

The directions of the incident and scatter wave normals are (Bahar 1981a)

$$\bar{n}^i = \sin \theta_o^i \cos \phi^i \bar{a}_x - \cos \theta_o^i \bar{a}_y + \sin \theta_o^i \sin \phi^i \bar{a}_z$$

$$\bar{n}^f = \sin \theta_o^f \cos \phi^f \bar{a}_y + \cos \theta_o^f \bar{a}_y + \sin \theta_o^f \sin \phi^f \bar{a}_z$$

Thus for backscatter $(\bar{n}^f = -\bar{n}^i) \theta_o^i = \theta_o^f = \theta_o, \phi^i = 0, \phi^f = \pi$.

$$\bar{n}^f = -\bar{n}^i = -\sin \theta_o \bar{a}_x + \cos \theta_o \bar{a}_y \quad (3.29a)$$

$$\bar{v} = 2k_0 \bar{n}^f \quad (3.29b)$$

For perfectly conducting surfaces the relative permittivity and permeability of the medium $y < h(x,z)$ is $|\epsilon_r| \rightarrow \infty$, $\mu_r \rightarrow 1$, thus $R_V \rightarrow 1$ and $R_H \rightarrow -1$ and the backscatter cross section $\langle \sigma_\infty^{PQ} \rangle_B$ (3.17) is given by (Bahar 1981b)

$$\langle \sigma_\infty^{PQ} \rangle_B = \delta_{PQ} \frac{\sec^4 \theta_o}{\sigma_{FS}^2} \exp - \left(\frac{\tan^2 \theta_o}{\sigma_{FS}^2} \right) \quad (3.30)$$

where δ_{PQ} is the Kronecker delta. The values for $W_m(v_x^-, v_z^-)$ (3.22), are evaluated numerically for $m = 2, 3$. (see Fig. 3.3a and 3.3b). In these plots $\beta = 1.5$, and $W_1 = W_T$ for $k > k_d$. As k_d decreases the number of significant terms in the expression for $\langle \sigma^{PQ} \rangle_R$ (3.18) increases. The individual terms in $\langle \sigma^{PQ} \rangle_R$ (i.e., $\langle \sigma^{PQ} \rangle_{Rm}$) can each be integrated numerically and summed to obtain $\langle \sigma^{PQ} \rangle_R$ or the integrands of the individual terms may be summed and numerically integrated once to give $\langle \sigma^{PQ} \rangle_R$. While the later procedure is more efficient, for the purpose of the illustrative examples presented here each individual term $\langle \sigma^{PQ} \rangle_{Rm}$ (3.19) is evaluated separately.

To provide a basis for comparing the full wave solutions with earlier results, the normalized backscatter cross sections are also evaluated using the analytical results recently derived by Brown (1978, 1980). Since his work is based on Burrows' perturbation theory (1967) and physical optics (Beckmann 1968), he specifies $k_d = 2\pi/\lambda_d$ (the wavenumber where spectral splitting is assumed to occur) on the basis of the characteristics of the small scale structure. Thus Brown concludes that his work

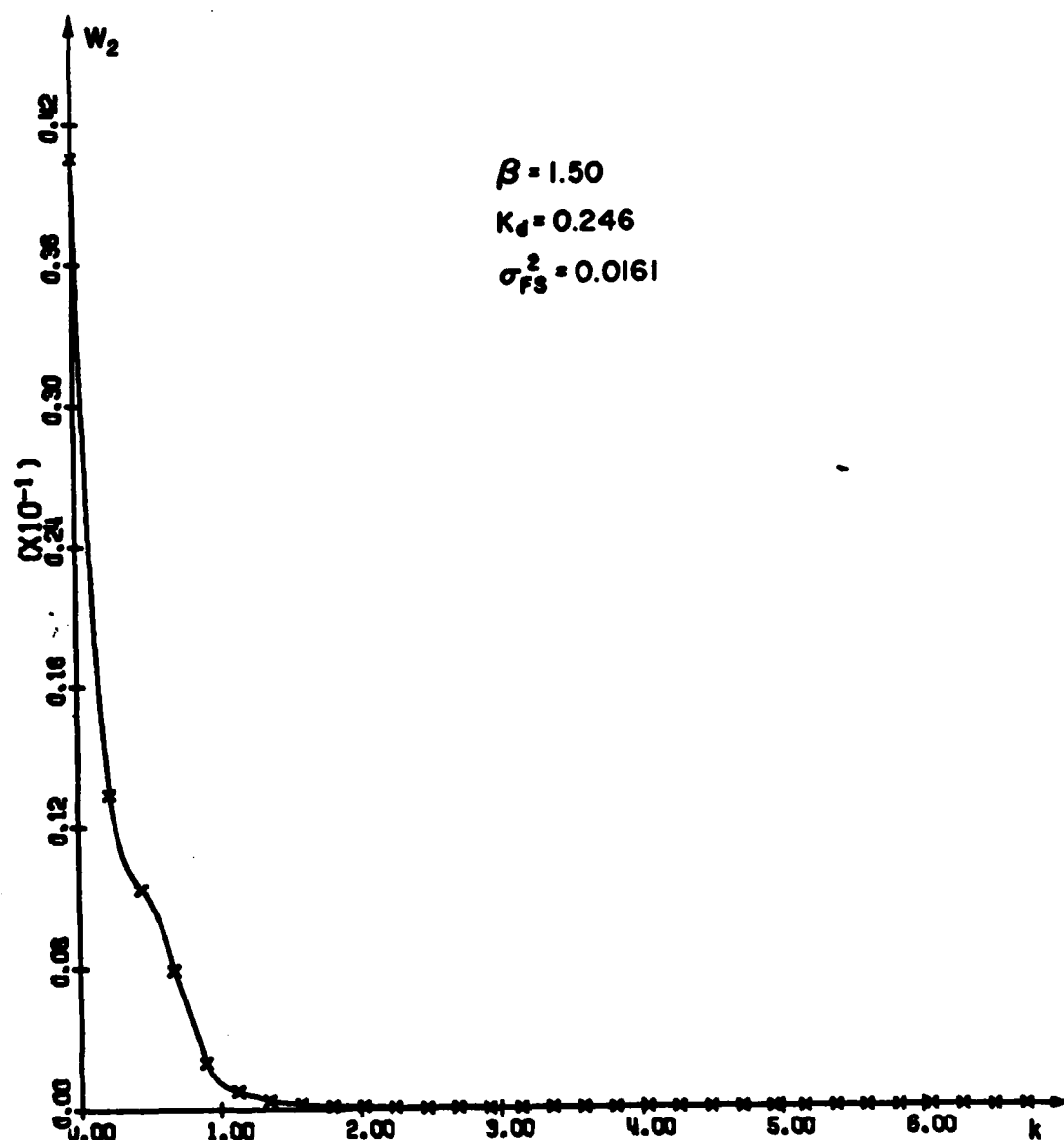


Fig. 3.3a. The function W_m (3.22) for $\beta = 1.5$

$m=2$

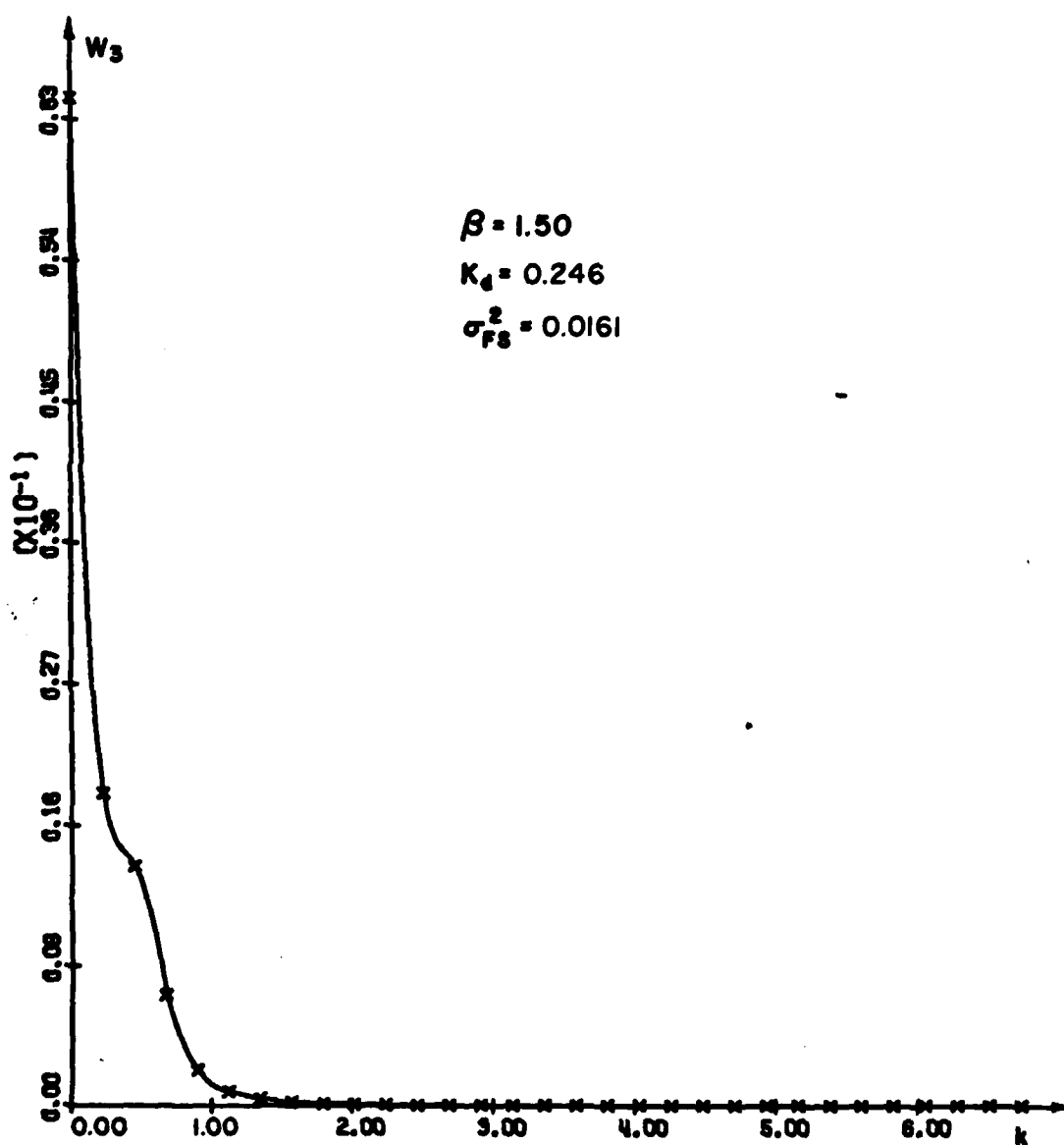


Fig. 3.3b. The function W_m (3.22) for $\beta = 1.5$

$m = 3$

"...clearly demonstrates the merit of choosing $\beta = 4k_o^2 \langle \bar{h}_R^2 \rangle$ as large as possible. On the other hand, β must be less than one in order to satisfy the basic criteria for the suitability of the perturbation technique, i.e., $4k_o^2 \langle \bar{h}_R^2 \rangle \ll 1$ ". Furthermore, on examining his illustrative examples, he notes that "It would appear from these numerical results that a proper choice of k_d should be based on the criterion $4k_o^2 \langle \bar{h}_R^2 \rangle \approx 0.1$ "... "Any attempt to draw a more positive conclusion about the choice of λ_d would have to address the basic question of the dividing lines between two types of scattering mechanisms," Brown goes on to say, "the results in this paper are based upon the assumption that the scattering is either physical optics (or geometric optics for k_o large enough) or small scale diffraction. Given this assumption, the criterion $4k_o^2 \langle \bar{h}_R^2 \rangle \approx 0.1$ seems reasonable." on the other hand, Brown (1978) notes that "...the concept of a truncated or filtered spectrum was first hypothesized by Hagfors (1966) in an attempt to explain lunar scattering data and the observed frequency dependence of near normal incidence scattering. More recently Tyler (1976) has attempted to definitize Hagfors' filter theory by basing the spectral truncation wavenumber on a criterion related to the radius of curvature of the large scale surface. Both these approaches base the point of spectral truncation upon a characteristic of the large scale structure, whereas, according to Brown, "it should be based upon the small scale structure i.e., $4k_o^2 \langle \bar{h}_R^2 \rangle \ll 1$." Finally, Brown points out that his results "...indicate a smooth transition between the geometric optics and Bragg scattering regimes which previously have been obtained in an ad hoc fashion."

Since the full wave solution, (3.1), accounts for both physical optics-specular scattering as well as Bragg scattering in a self-consistent manner, it is used here to explore the basic question of an apparent dividing line between the two types of scattering mechanisms.

In Figs. 3.4a, 3.4b, and 3.4c, the normalized backscatter cross section $\langle \sigma^{PQ} \rangle$ based on Brown's results are presented for $\beta = 0.1$ (corresponding to $\sigma_{FS}^2 = (\bar{\zeta}_{gt}^2) = 0.0224$ and $k_d = 0.95 \text{ (cm)}^{-1}$). These figures show $\langle \sigma^{PQ} \rangle = \sigma_{PQ}^0$ (the total), $\langle \sigma^{PQ} \rangle_F = [\sigma_{PQ}^0]_0$ and $\langle \sigma^{PQ} \rangle_R = [\sigma_{PQ}^0]_1$. Figs. 3.4a and 3.4b for $\langle \sigma^{VV} \rangle$ and $\langle \sigma^{HH} \rangle$ differ slightly from the results presented by Brown (1978) for $0 \leq \theta_o \leq 70^\circ$ since he approximates Γ_{pp}^2 , (h_{Fx}, h_{Fz}) (related to S^{PQ}) by its zero slope approximation $\Gamma_{pp}^2(0,0)$ and replaces the shadow function R (P_2 in the notation used here) by unity. These approximations enable Brown to analytically convert the two dimensional integrals into one dimensional integrals which he evaluates numerically. The main differences are near grazing angles ($\theta_o > 80^\circ$ not shown in Brown's work) where shadowing becomes significant. Moreover, since $\Gamma_{VH}^2(0,0) = 0$ he does not provide numerical data for the cross-polarized backscatter cross section $\langle \sigma^{VH} \rangle$ (Fig. 3.4c). In view of Brown's comments regarding the optimal choice of β (0.1) and the sensitivity to his numerical results to changes in β , no other results based on Brown's work are presented here. In Figs. 3.5a, 3.5b, and 3.5c, the corresponding results ($\beta = 0.1$) based on the full wave solutions (3.15) are presented. In each of these figures $\langle \sigma^{PQ} \rangle$ (the total backscatter cross section) $\langle \sigma^{PQ} \rangle_F$ (3.16), (the cross section associated with the filtered surfaces) as well as $\langle \sigma^{PQ} \rangle_{R1}$ and $\langle \sigma^{PQ} \rangle_{R2}$ (3.19) (the cross sections associated with h_R) are

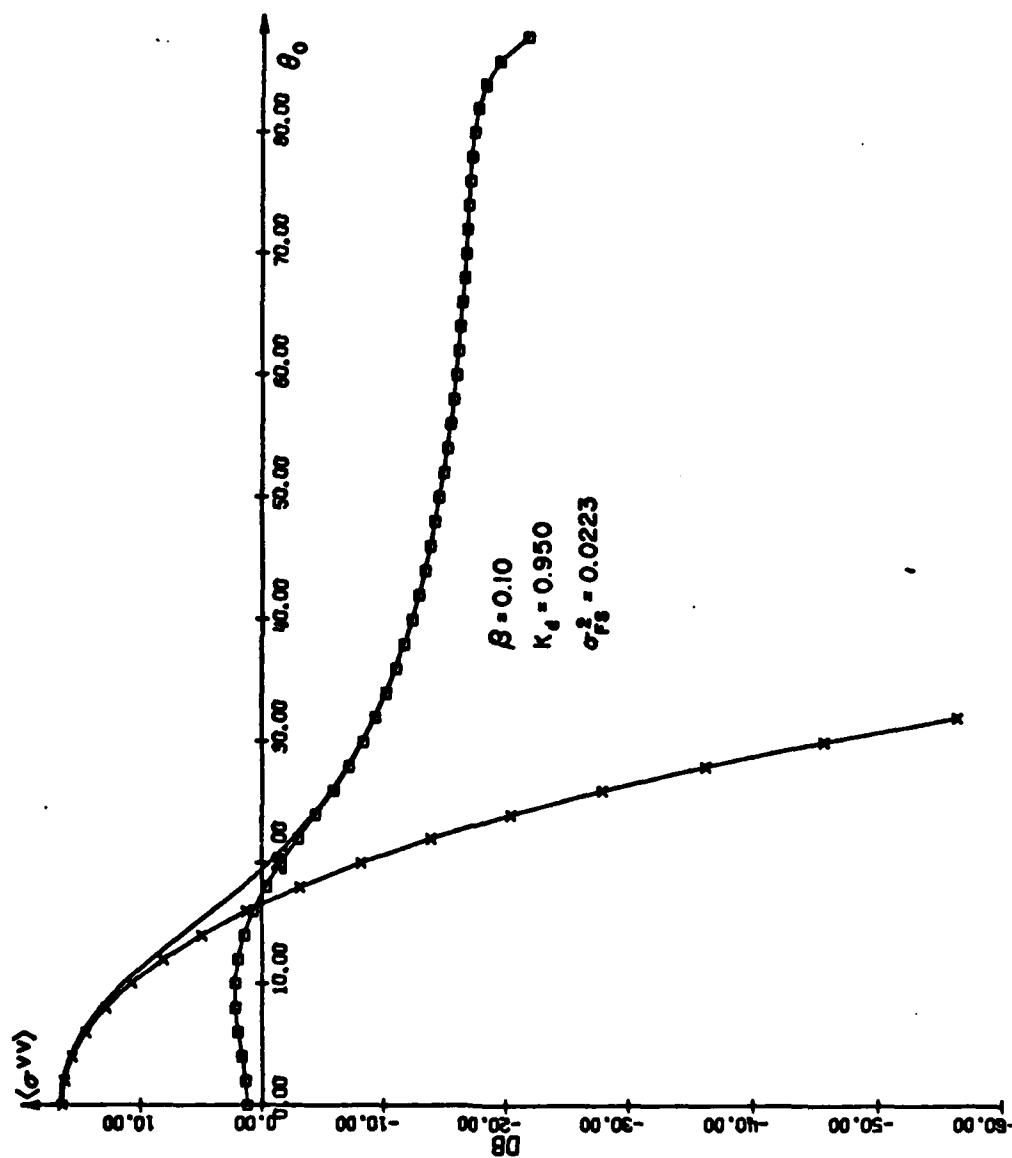


Fig. 3.4a. Backscatter cross sections for rough surface (3.14) using Brown's formulation (1978, 1980) for $\beta = 0.1$ (— $\langle \sigma^{PQ} \rangle$ (total), $\times \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$), $\langle \sigma^{VV} \rangle$

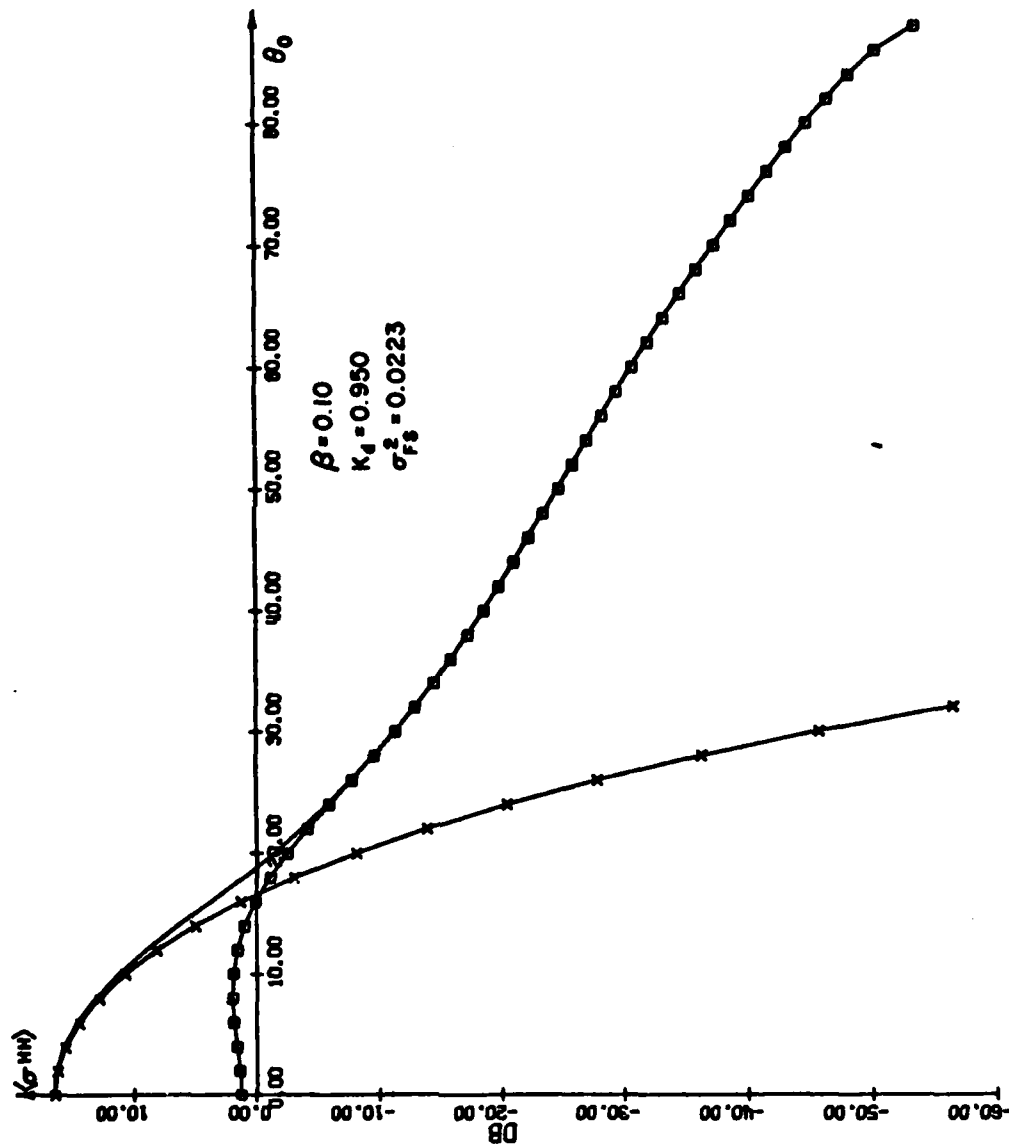


Fig. 3.4b. Backscatter cross sections for rough surface (3.14) using Brown's formulation (1978, 1980) for $\beta = 0.1$ (— $\langle \sigma^{PQ} \rangle$ (total), $\times \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$). $\langle \sigma^{HH} \rangle$

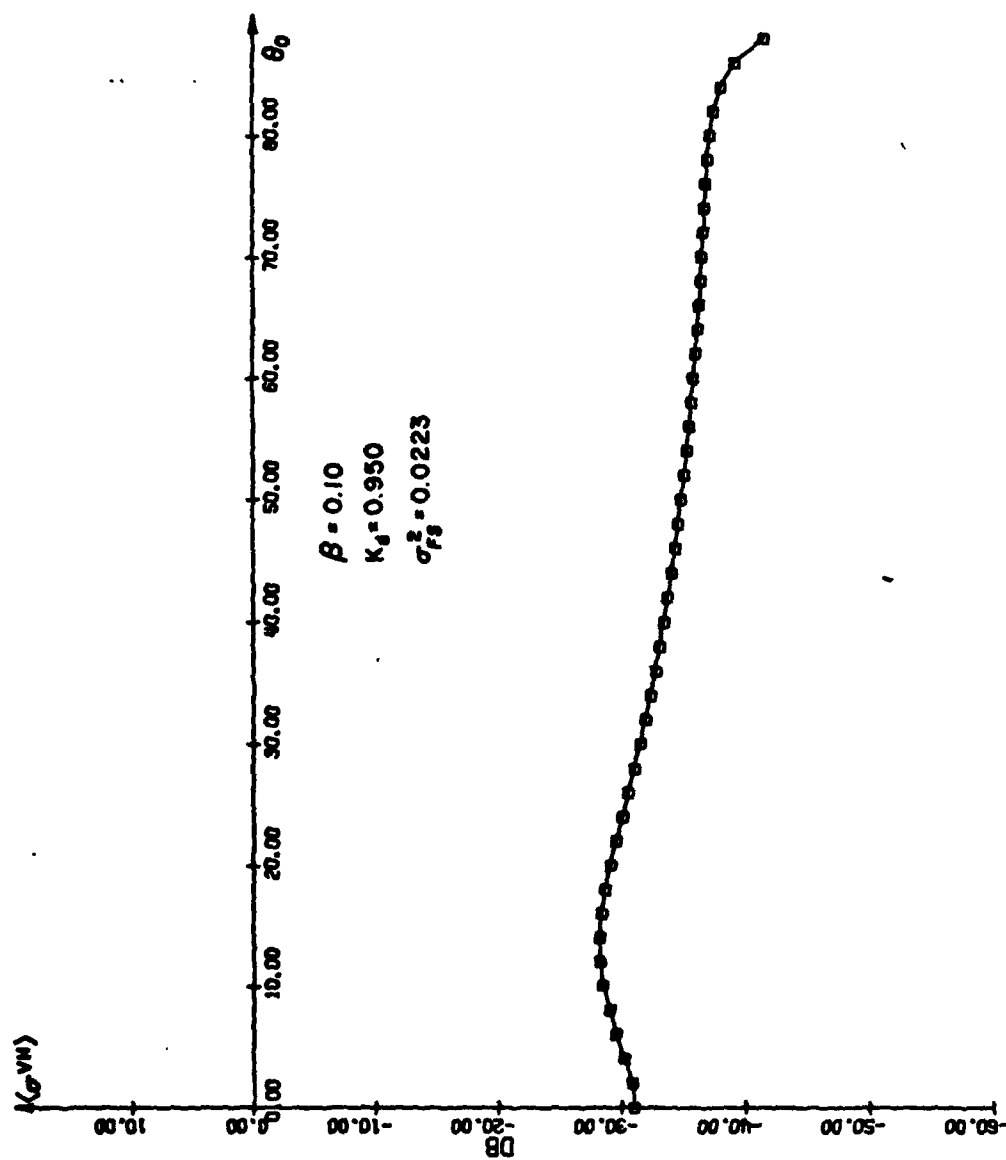


Fig. 3.4c. Backscatter cross sections for rough surface (3.14) using Brown's formulation (1978, 1980) for $\beta = 0.1$ (--- $\langle \sigma^{PQ} \rangle$ (total), $X \langle \sigma^{PQ} \rangle_F$, $\sigma \langle \sigma^{PQ} \rangle_{R1}$). $\sigma^{VH} \langle \sigma \rangle$

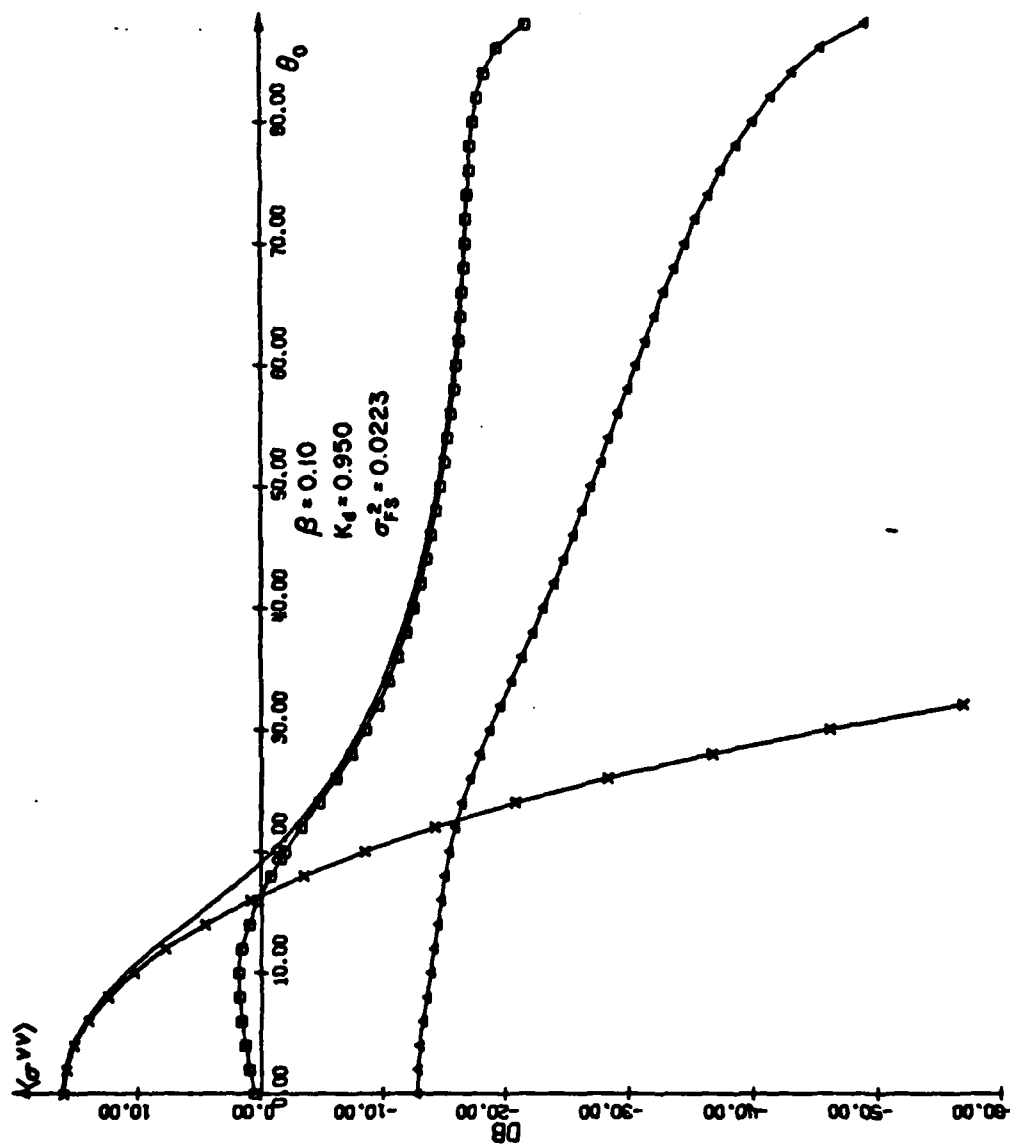


Fig. 3.5a. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.1$
 (— $\langle \sigma^{PQ} \rangle$ (total), X $\langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, $\langle \sigma^{VV} \rangle$)

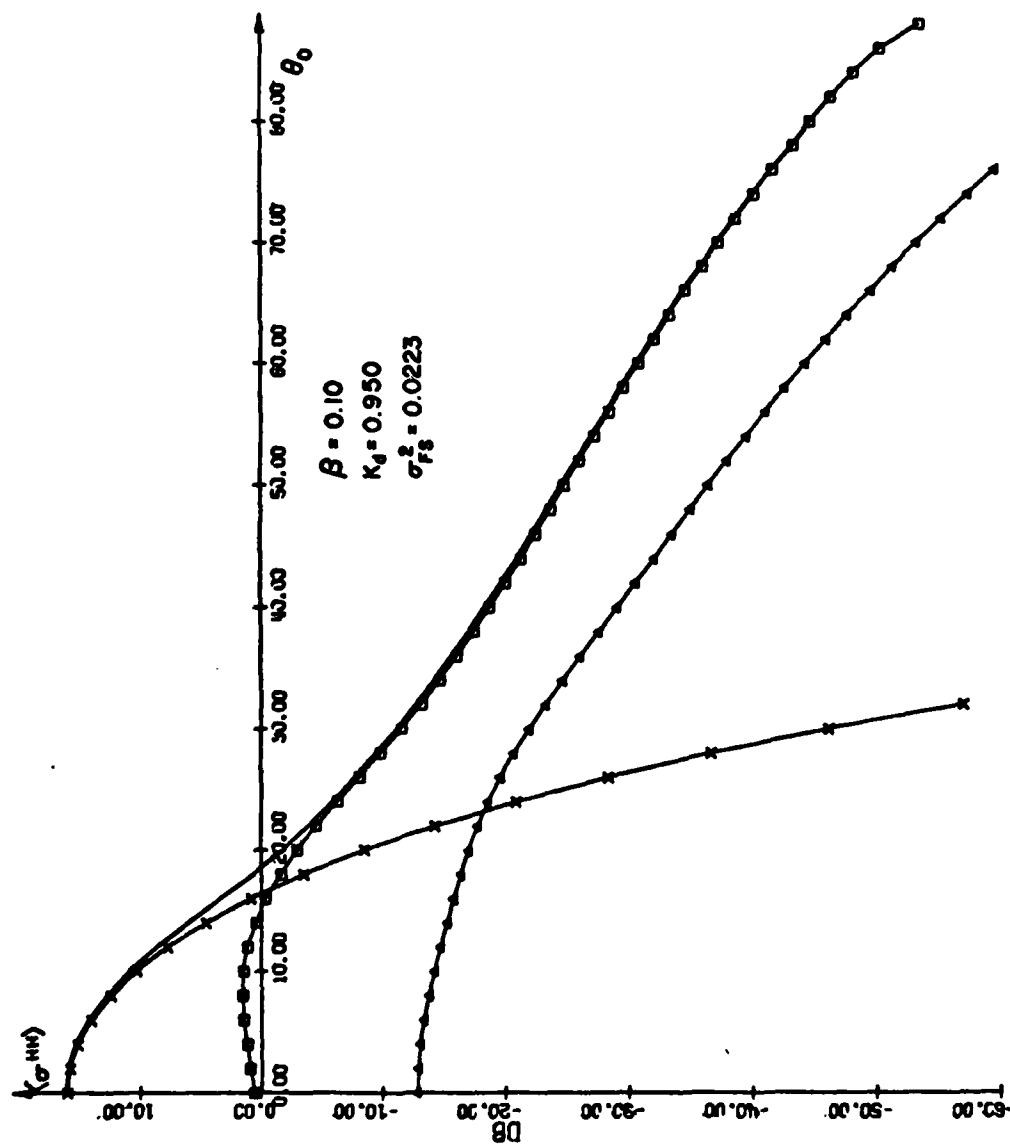


Fig. 3.5b. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.1$
 (— $\langle \sigma^{PQ} \rangle$ (total), $X \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, $\circ \langle \sigma^{HH} \rangle$).

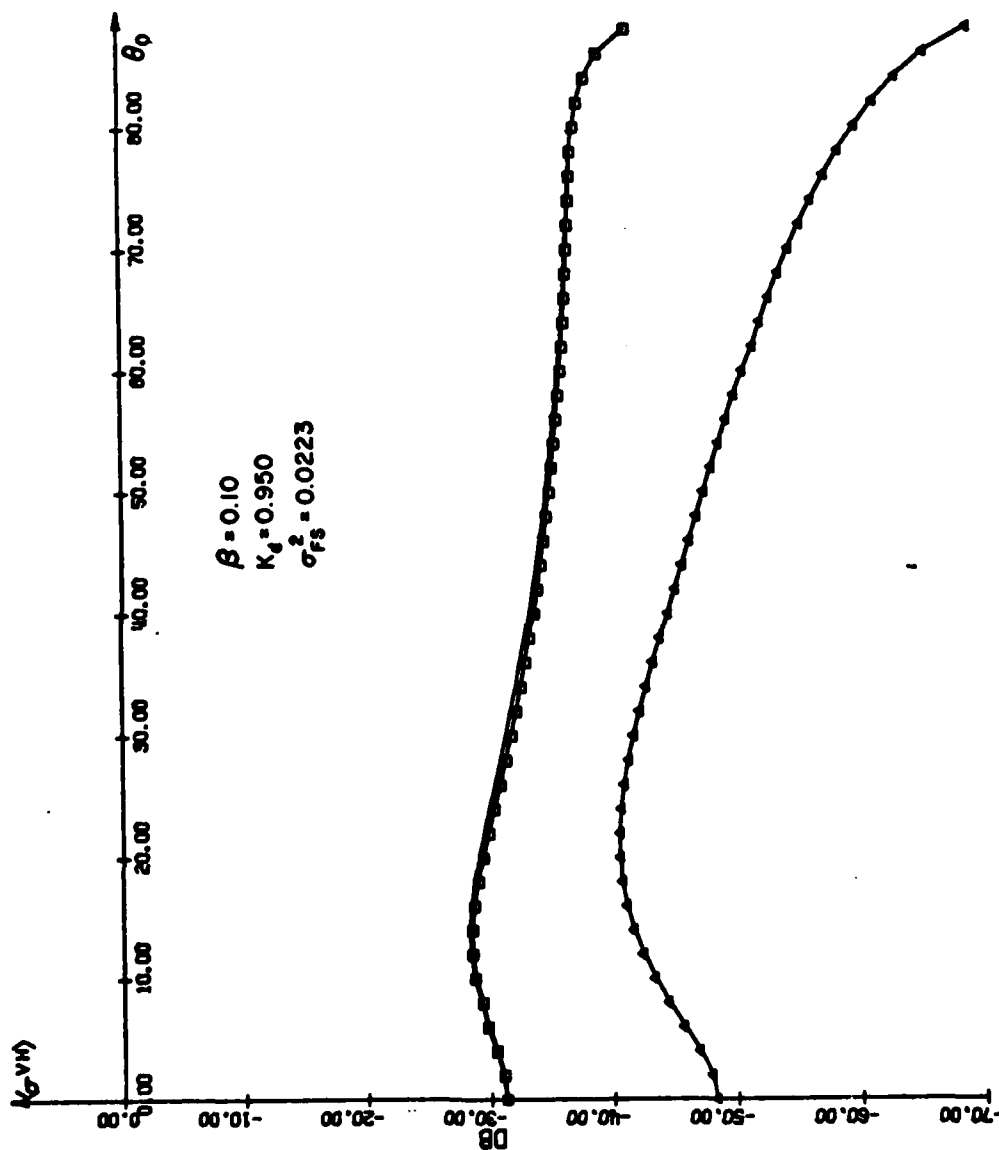


Fig. 3.5c. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.1$
 (— $\langle \sigma^{PQ} \rangle$ (total), $X \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, $\langle \sigma^{VH} \rangle$).

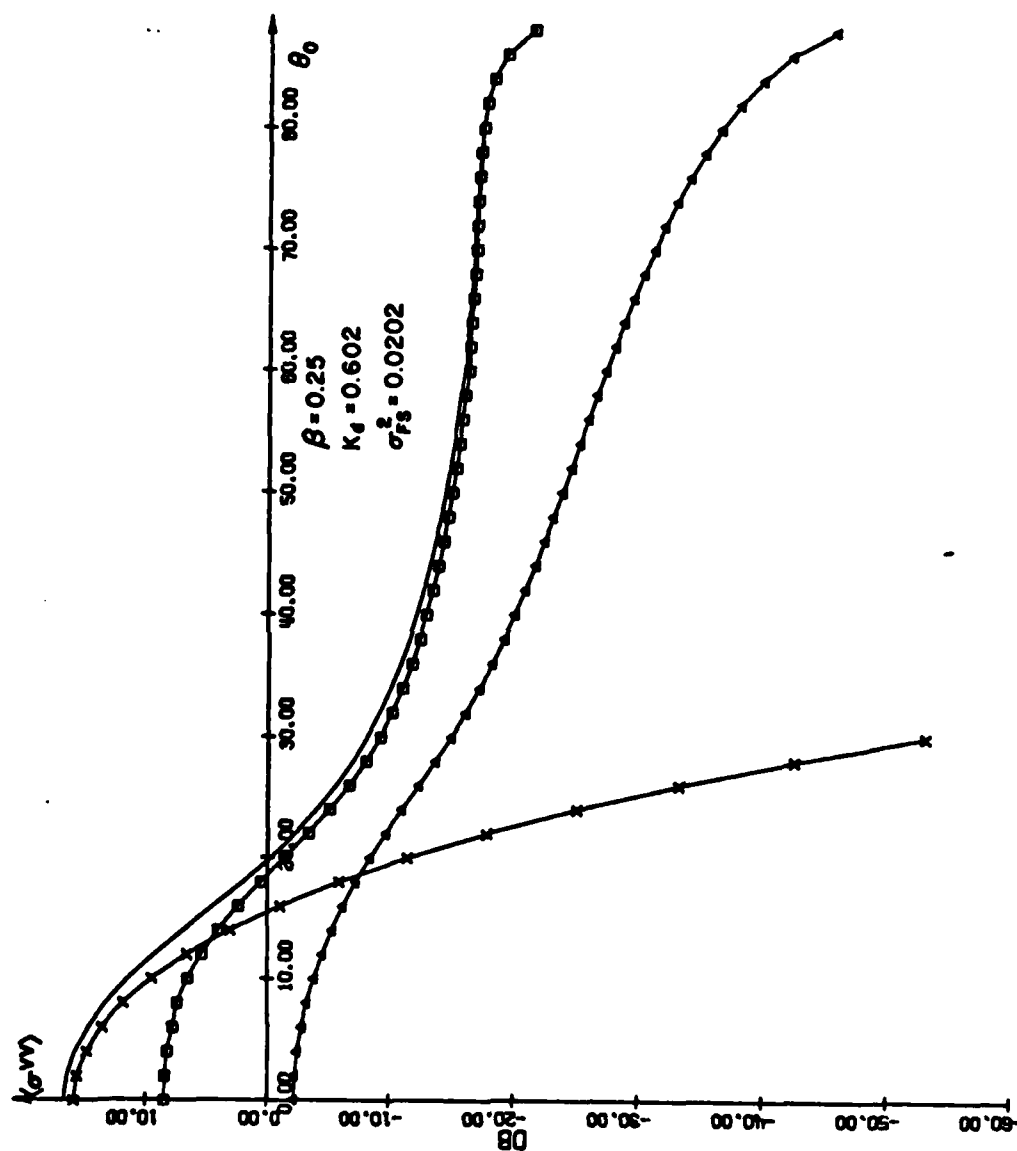


Fig. 3.6a. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.25$
 (— $\langle \sigma^{PQ} \rangle$ (total), $X \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, $\circ \langle \sigma^{VV} \rangle$).

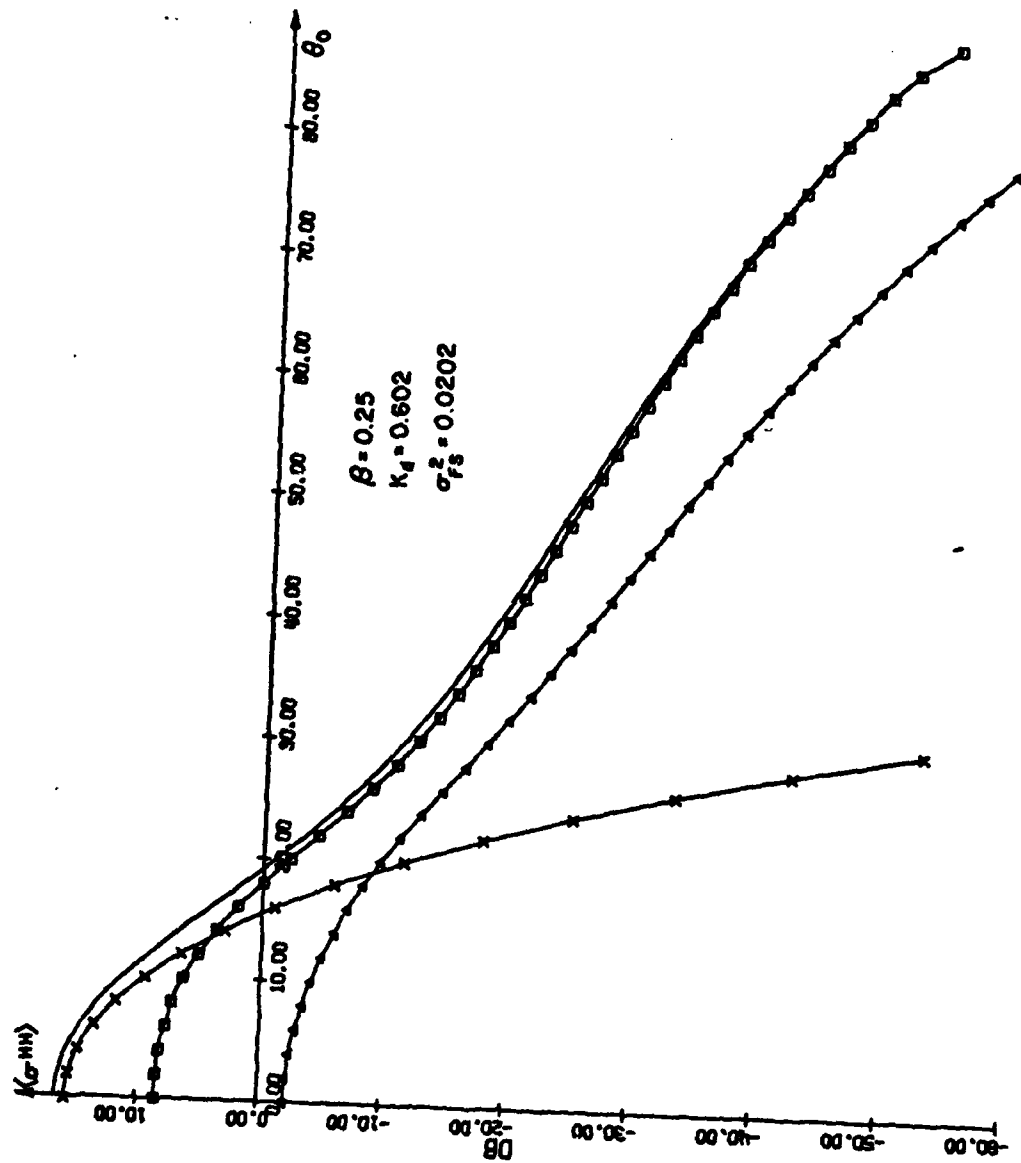


Fig. 3.6b. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.25$
 (— $\langle \sigma^{PQ} \rangle$ (total), $X \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, $\times \langle \sigma^{HH} \rangle$).

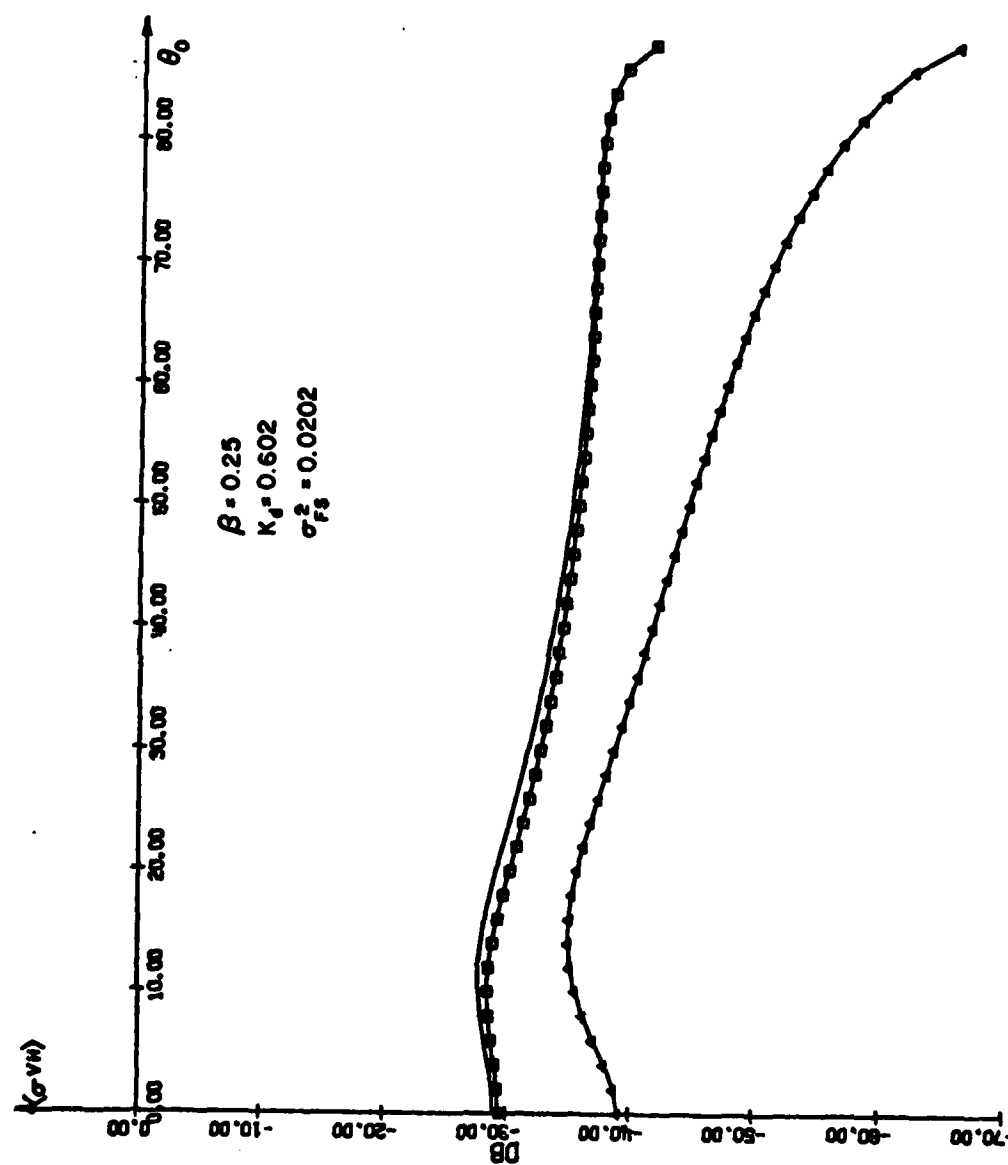


Fig. 3.6c. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.25$
 (— $\langle \sigma^{PQ} \rangle$ (total), $X \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, $\langle \sigma^{VH} \rangle$).

presented. For $\beta = 0.1$, $\langle \sigma^{RQ} \rangle_{Rm}$ for $m \geq 3$ are not shown since they are too small to affect the total cross section $\langle \sigma^{PQ} \rangle$. Furthermore, since $|\chi^R(\vec{v} \cdot \vec{n}_s)|^2 = e^{-\beta} \approx 1$, $\exp(-v \frac{2}{y} \langle h_R^2 \rangle) \approx 1$, and $\langle \sigma^{PQ} \rangle_{R2}$ is small compared to $\langle \sigma^{PQ} \rangle_{R1}$, these results are very similar to those based on Brown's work. In Figs. 3.6a, 3.6b, and 3.6c through 3.9a, 3.9b and 3.9c, the scattering cross sections based on the full wave solutions are presented for $\beta = 0.25$, 0.5, 1.0 and 1.5. In Figs. 3.9a, 3.9b, and 3.9c, the term $\langle \sigma^{PQ} \rangle_{R3}$ is also presented since it becomes significant for $\beta \geq 1.5$. In Fig. 3.10, $\langle \sigma^{VV} \rangle$ is presented for $\beta = 2.0$ to demonstrate the importance of the term $\langle \sigma^{VV} \rangle_{R3}$ as β increases above the value of $\beta = 1.5$.

From the results based on the full wave solutions (Figs. 3.5 through 3.10), one notices that while the individual contributions to the total backscatter cross sections $\langle \sigma^{PQ} \rangle$ are very sensitive to the value of β (which determines k_d) the value for the total cross section varies slightly as β increases from 0.1 to 1.0 and insignificantly for $\beta \geq 1.0$. This latter result is shown explicitly in Figs. 3.11a, 3.11b, and 3.11c, where only the total backscatter cross sections $\langle \sigma^{PQ} \rangle$ are plotted for $\beta = 0.1$, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, and 2.0. It was not considered necessary to evaluate $\langle \sigma^{PQ} \rangle$ for $\beta < 0.1$ since these results would practically duplicate those provided by Brown (1978). In Table 3.1, the computed values for k_d (3.26) and σ_{FS}^2 (3.27) corresponding to values of β from 0.1 to 2.0 are listed.

Clearly the above numerical results indicate that from the point of view of both accuracy (independence of the results on the choice of β or

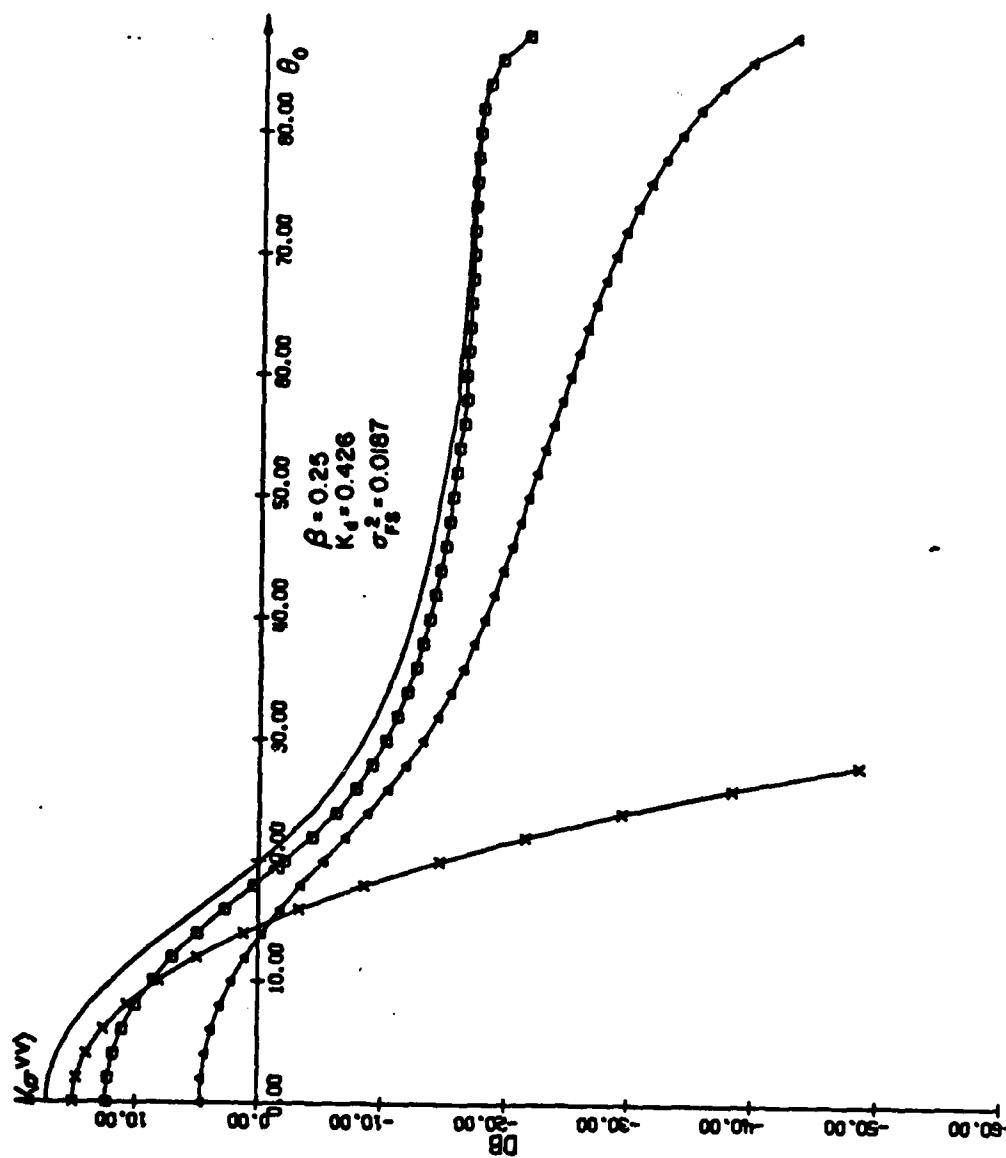


Fig. 3.7a. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.5$
 (— $\langle \sigma^2_{PQ} \rangle$ (total), $X \langle \sigma^2_{PQ} \rangle_F$, $\square \langle \sigma^2_{PQ} \rangle_{R1}$, $\Delta \langle \sigma^2_{PQ} \rangle_{R2}$, $\langle \sigma^2_{PQ} \rangle_{VW}$).

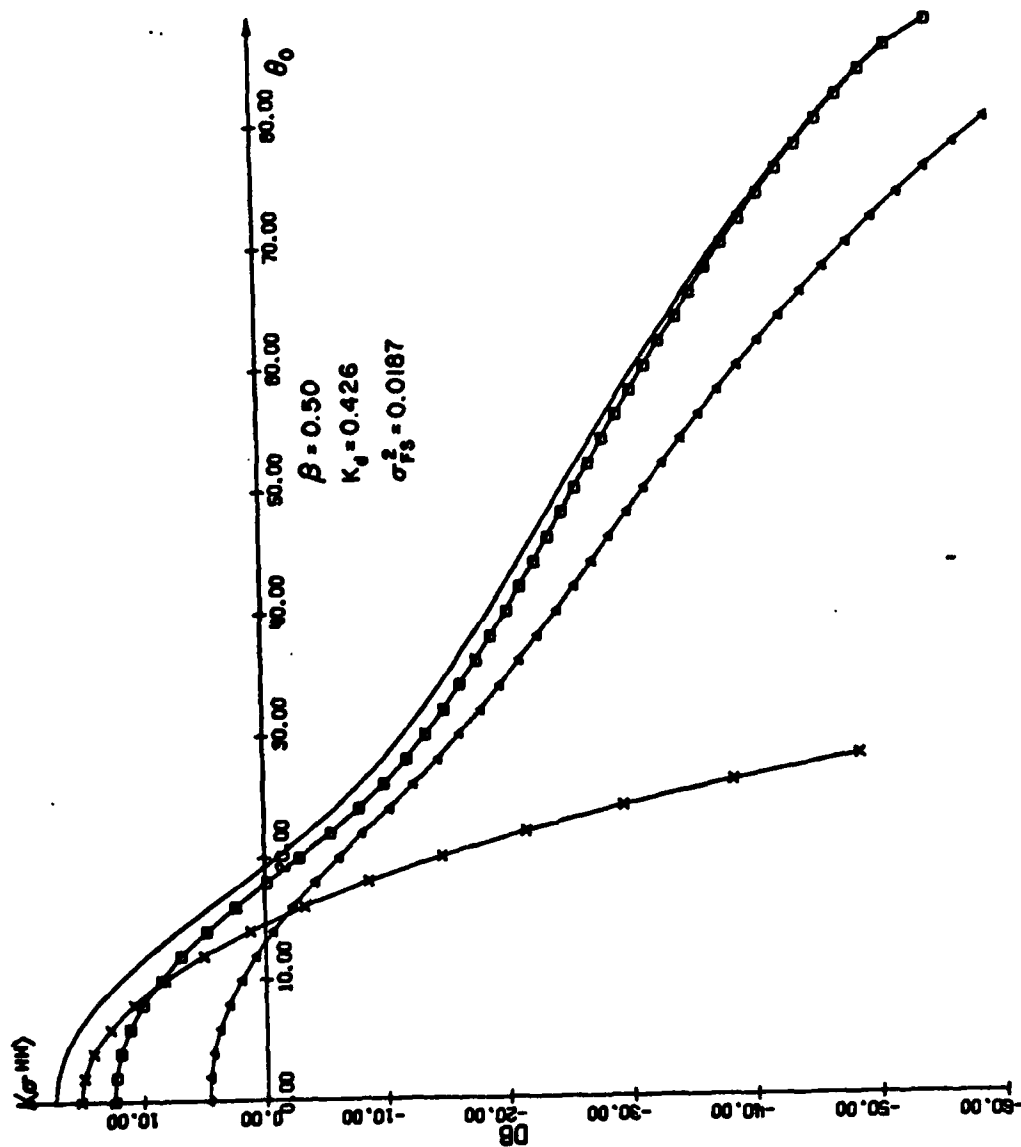


Fig. 3.7b. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.5$
 (— $\langle \sigma^{PQ} \rangle$ (total), $X \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, $\circ \langle \sigma^{HH} \rangle$).

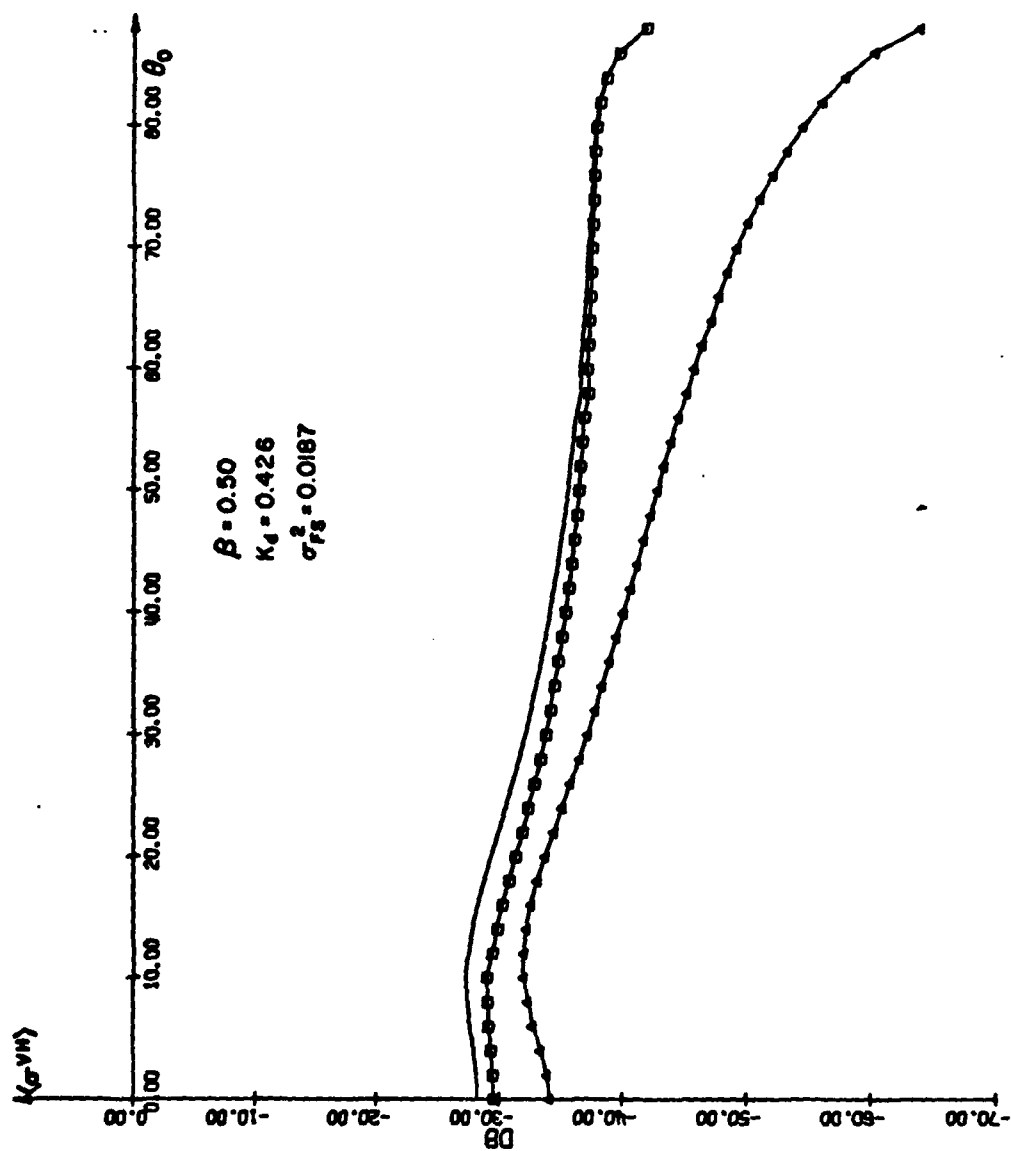


Fig. 3.7c. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.5$
 (— $\langle \sigma_{PQ} \rangle$ (total), X $\langle \sigma_F \rangle$, □ $\langle \sigma_{PQ} \rangle_{R1}$, Δ $\langle \sigma_{PQ} \rangle_{R2}$, $\langle \sigma_{VH} \rangle$).

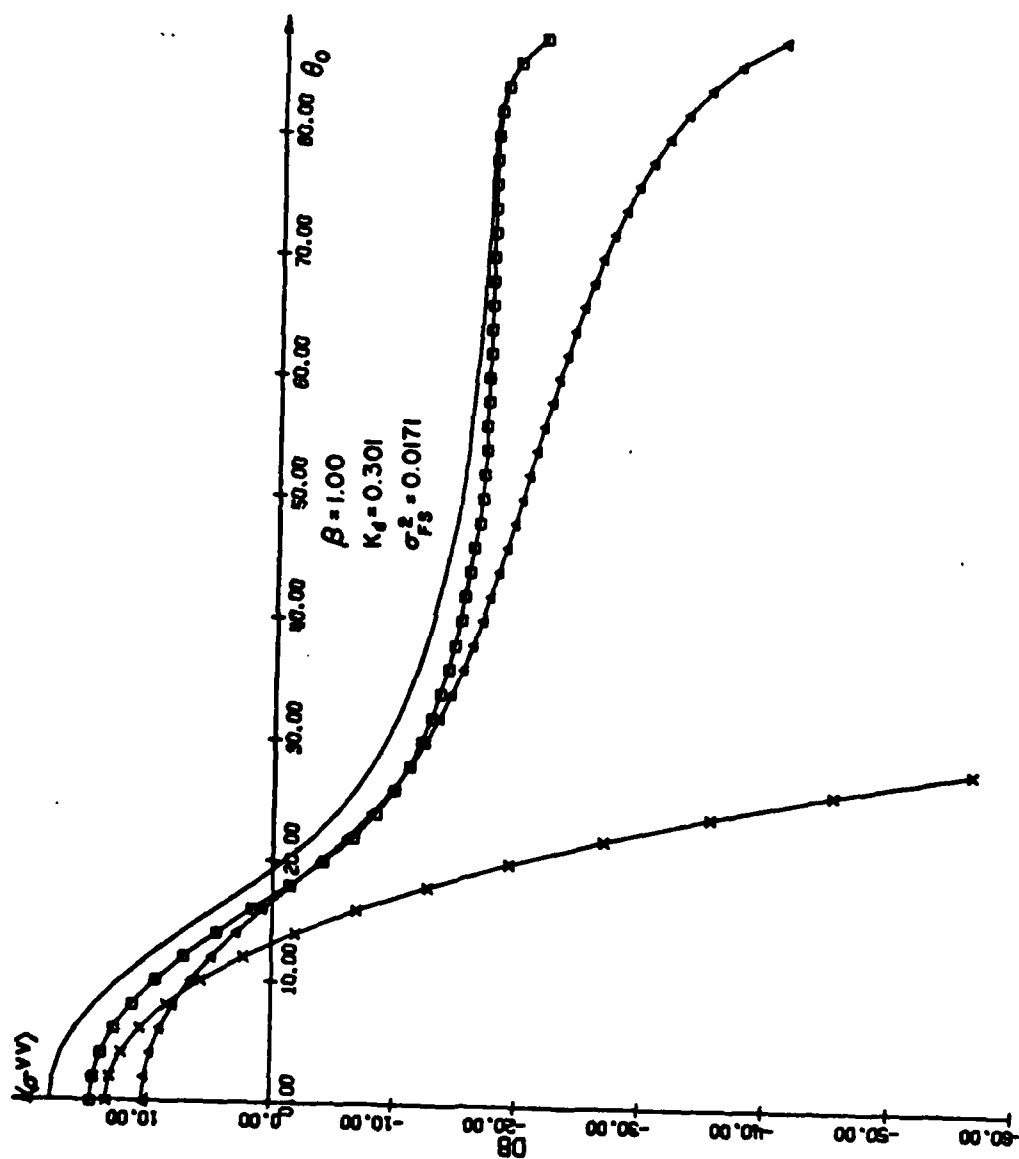


Fig. 3.8a. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 1.0$
 (— $\langle \sigma^{PQ} \rangle$ (total), $\times \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, $\nabla \langle \sigma^{PQ} \rangle$).

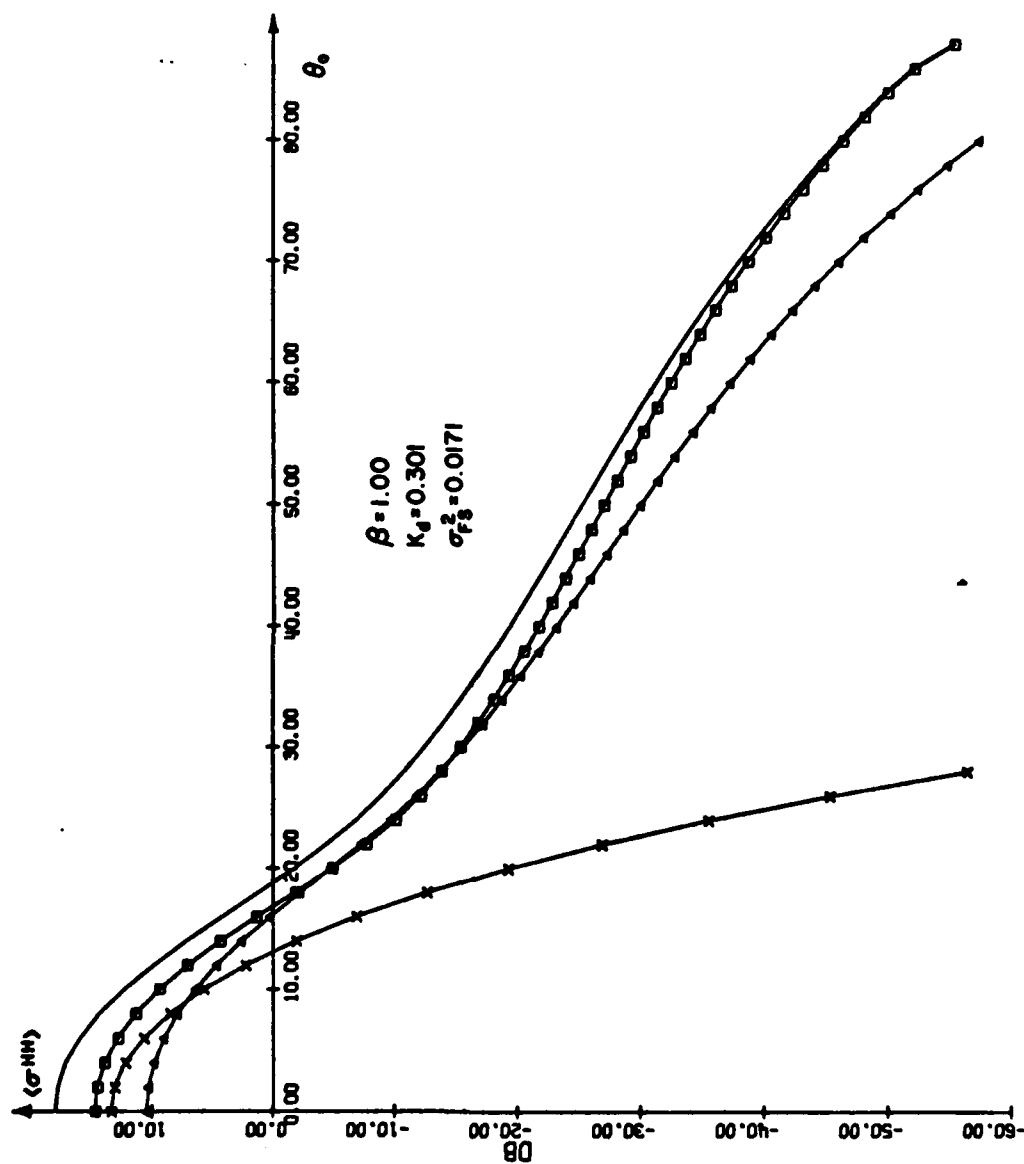


Fig. 3.8b. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 1.0$
 (— $\langle \sigma_{PQ}^{PQ} \rangle_{\text{total}}$, $X \langle \sigma_{PQ}^{PQ} \rangle_{R1}$, $\Delta \langle \sigma_{PQ}^{PQ} \rangle_{R2}$, $\langle \sigma_{HH} \rangle$).

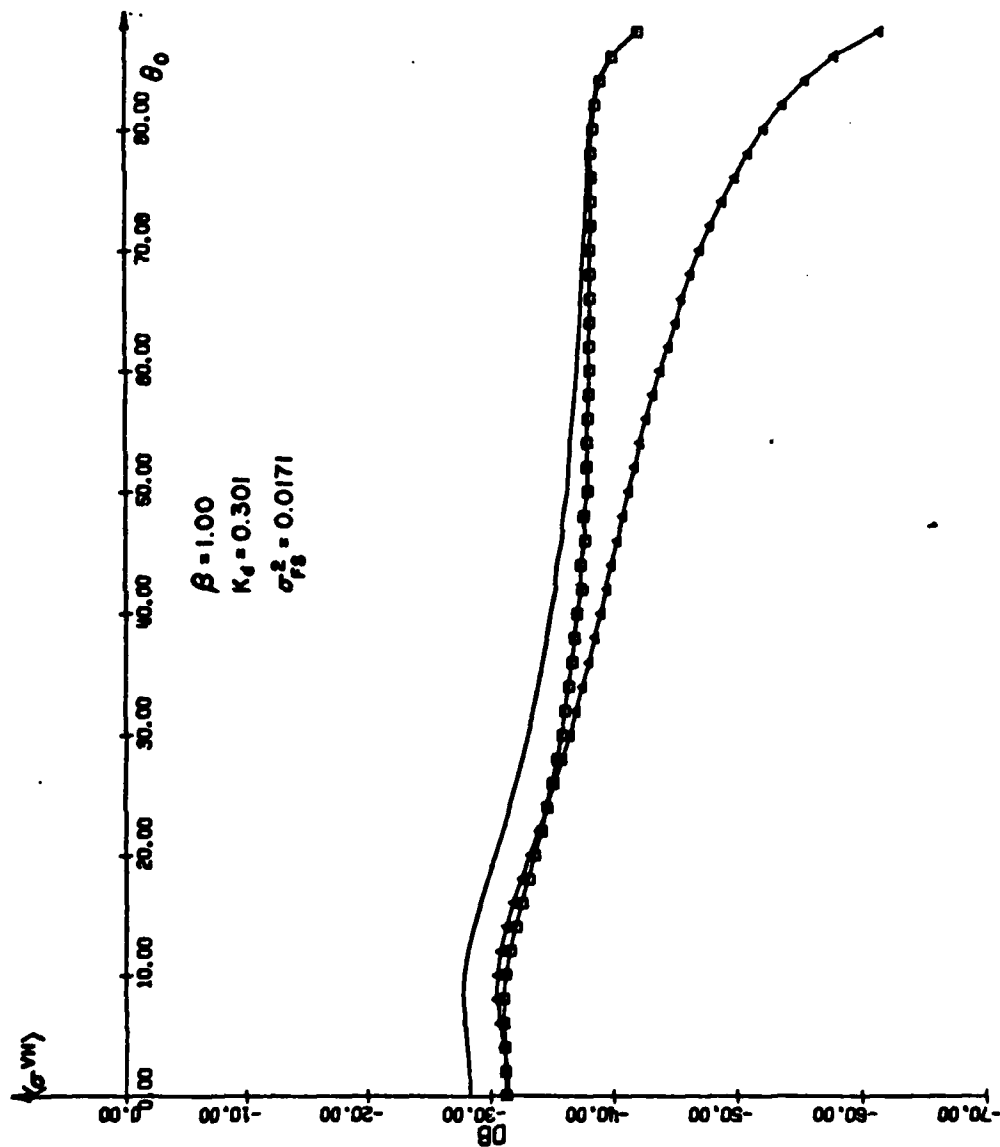


Fig. 3.8c. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 1.0$
 (— $\langle \sigma^{PQ} \rangle_{\text{total}}$, \square $\langle \sigma^{PQ} \rangle_F$, Δ $\langle \sigma^{PQ} \rangle_{R2}$, \circ $\langle \sigma^{VH} \rangle$).

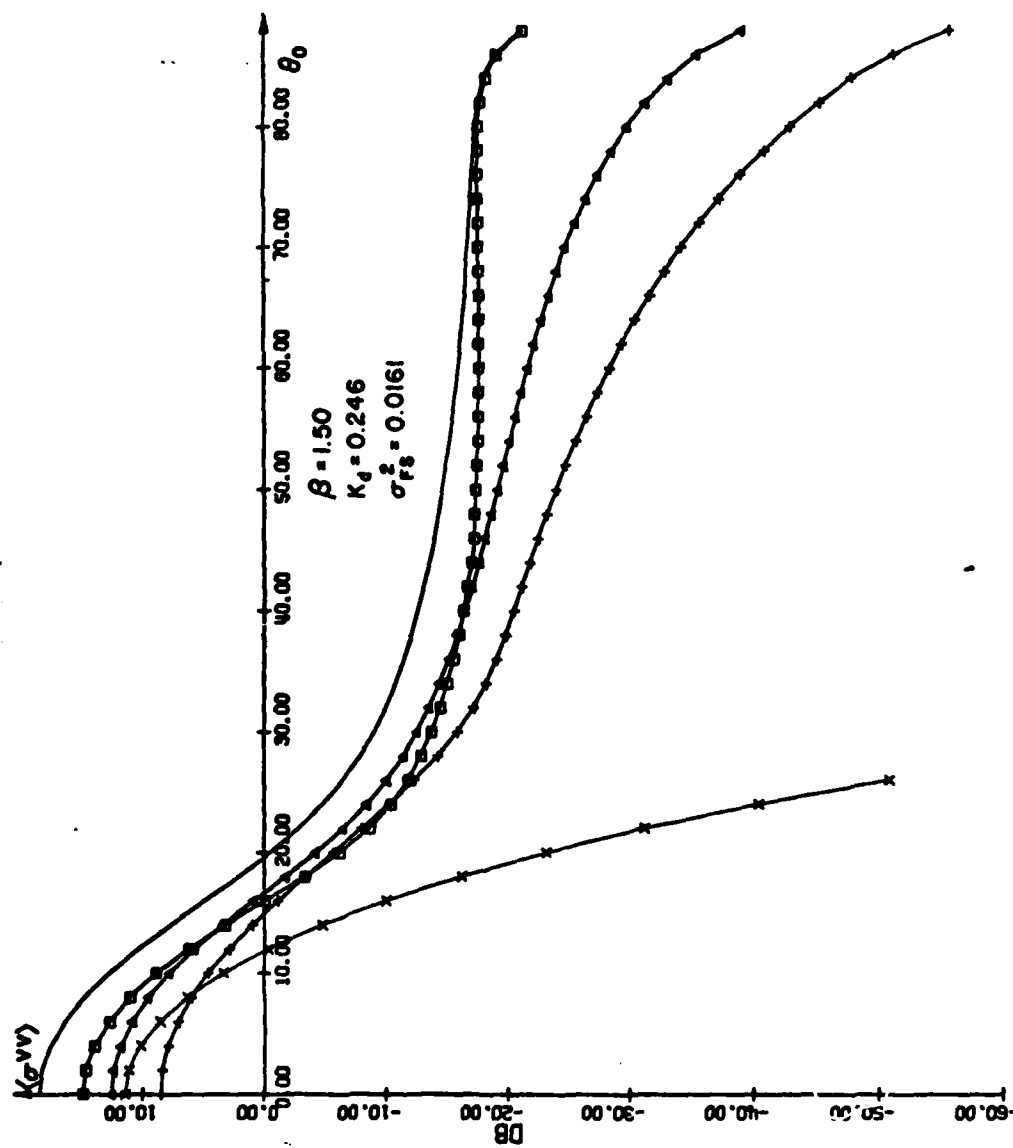


Fig. 3.9a. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 1.5$
 (---) $\langle \sigma^{PQ} \rangle$ (total), X $\langle \sigma^{PQ} \rangle_F$, \square $\langle \sigma^{PQ} \rangle_{R1}$, \triangle $\langle \sigma^{PQ} \rangle_{R2}$, \circ $\langle \sigma^{PQ} \rangle_{R3}$.

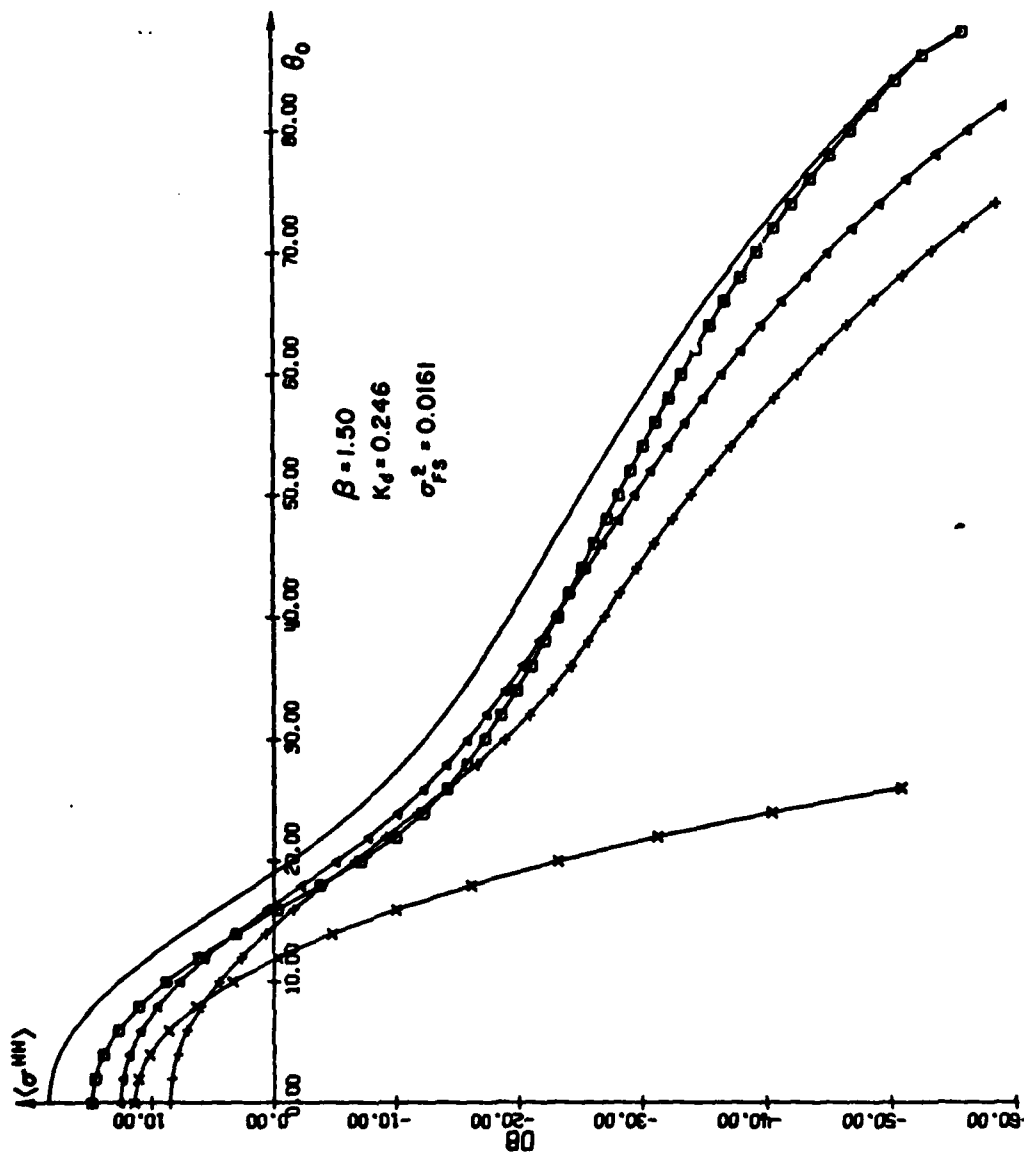


Fig. 3.9b. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 1.5$
 (--- $\langle \sigma^{PQ} \rangle$ (total), X $\langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, + $\langle \sigma^{PQ} \rangle_{R3}$, $\langle \sigma^{HH} \rangle$).

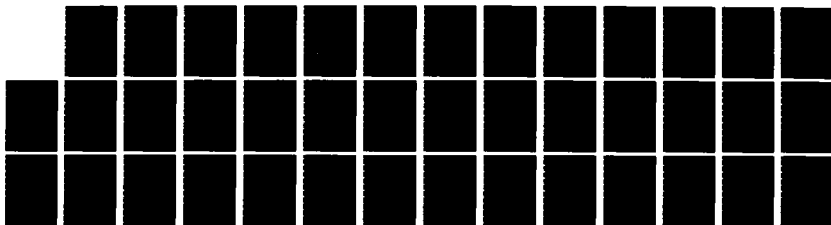
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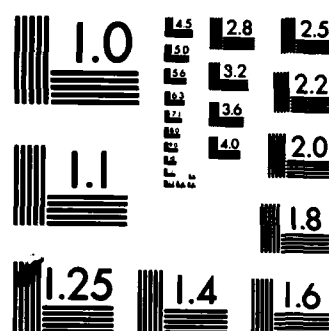
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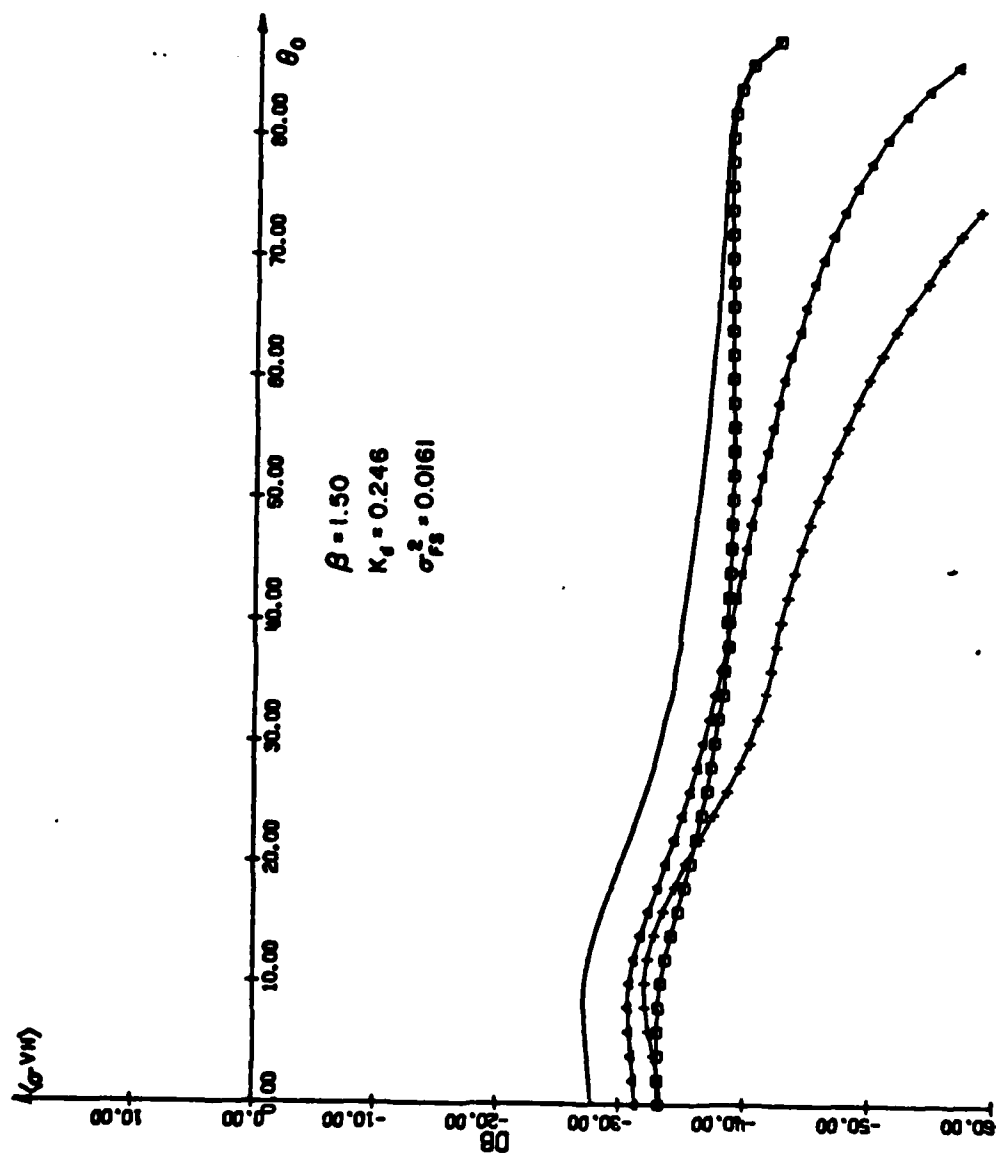


Fig. 3.9c. Backscatter cross sections for rough surface (3.14) using full wave formulation (3.15) for $\beta = 1.5$
 (— $\langle \sigma^{PQ} \rangle$ (total), $X \langle \sigma^{PQ} \rangle_F$, $\square \langle \sigma^{PQ} \rangle_{R1}$, $\Delta \langle \sigma^{PQ} \rangle_{R2}$, $+ \langle \sigma^{PQ} \rangle_{R3}$, $\circ \langle \sigma^{PQ} \rangle_{VH}$).

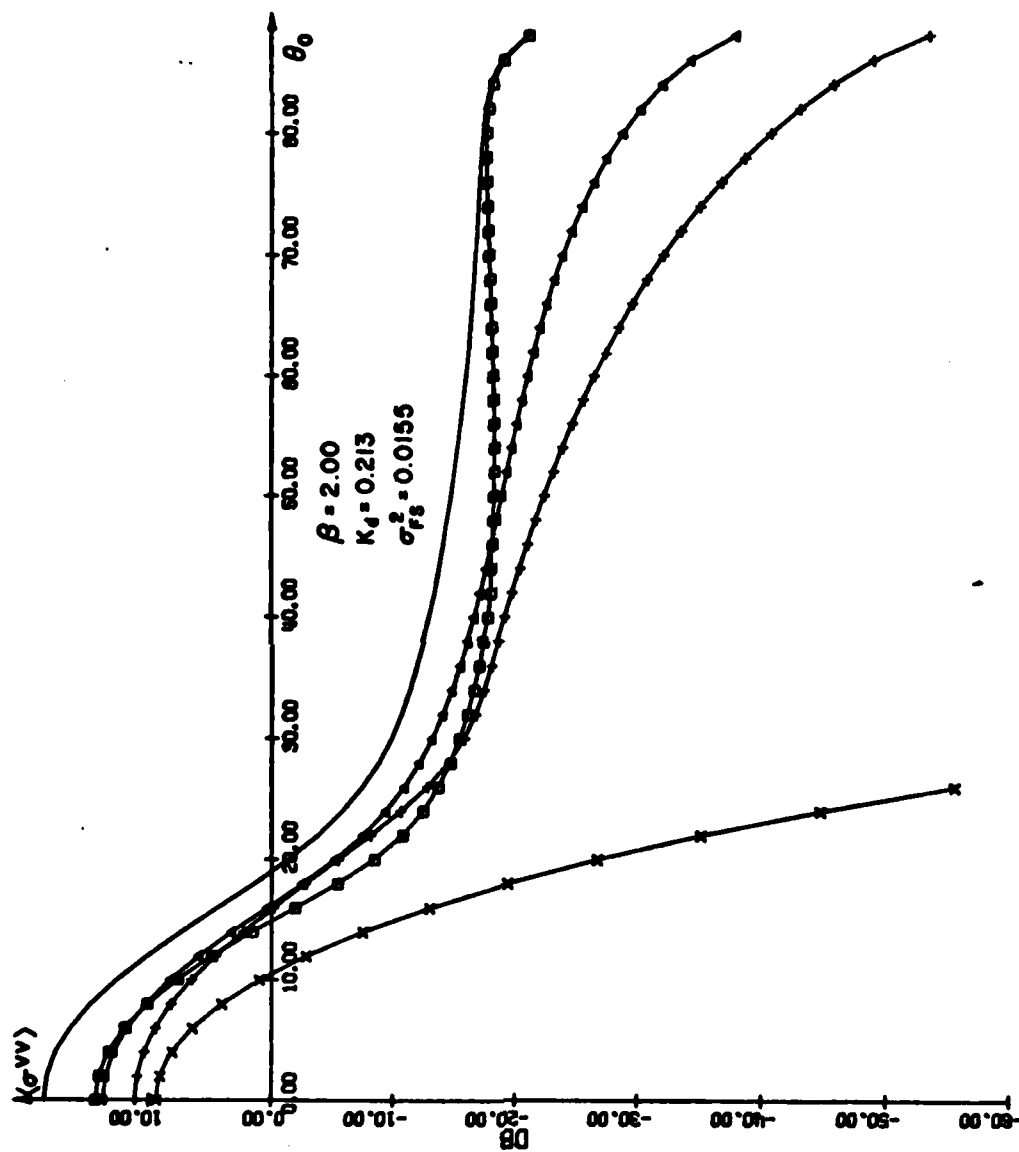


Fig. 3.10. Backscatter cross sections $\langle \sigma^V \rangle$ for rough surface (3.14) using full wave formulation (3.15) for $\beta = 2.0$
 (— $\langle \sigma^V \rangle$ (total), X $\langle \sigma^V \rangle_F$, \square $\langle \sigma^V \rangle_{R1}$, Δ $\langle \sigma^V \rangle_{R2}$, \diamond $\langle \sigma^V \rangle_{R3}$.

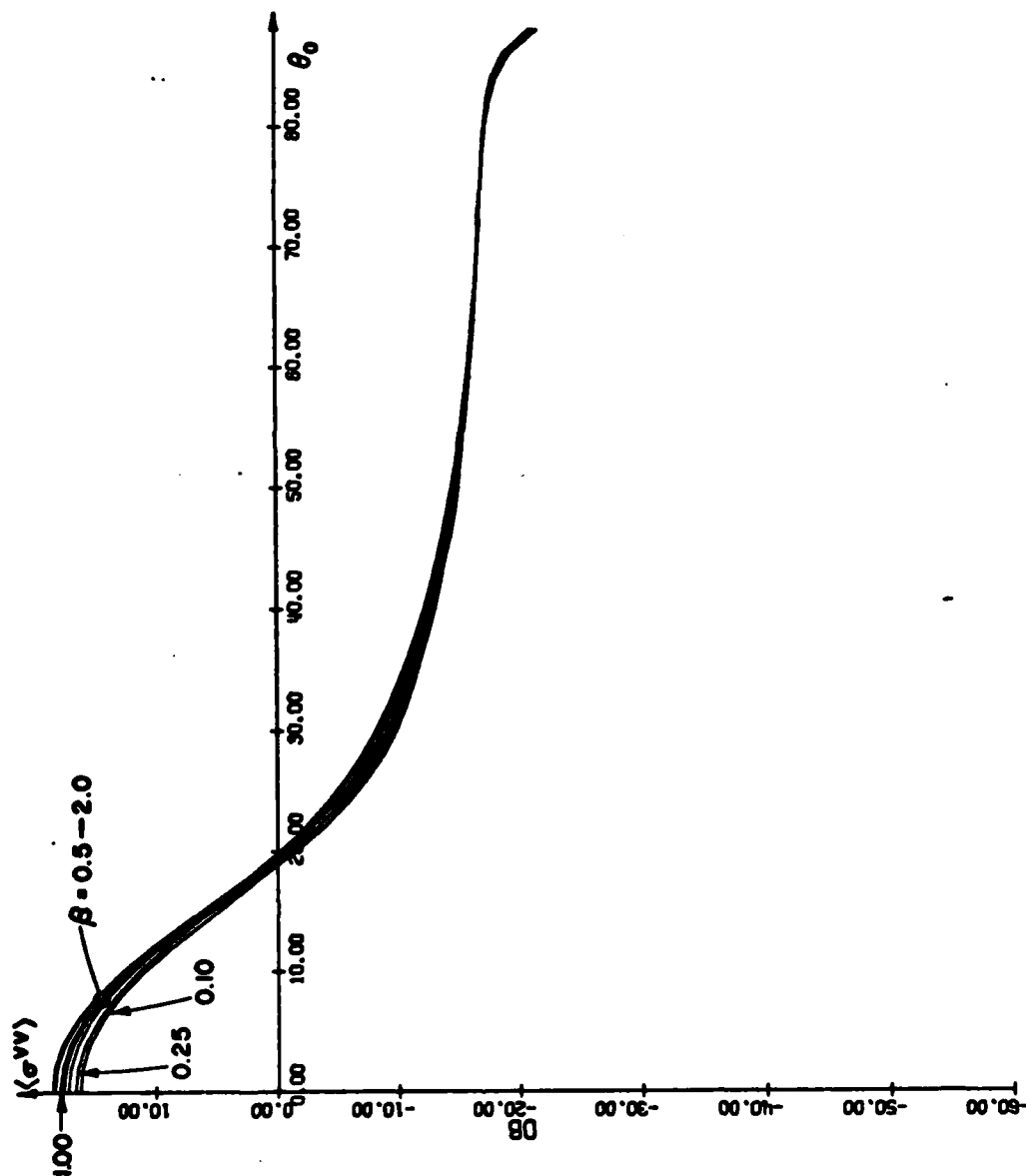


Fig.3.11a. Backscatter cross sections $\langle \sigma^{PQ} \rangle$ (total) for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.1, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75$, and $\beta = 2.0$. $\langle \sigma^{VV} \rangle$.

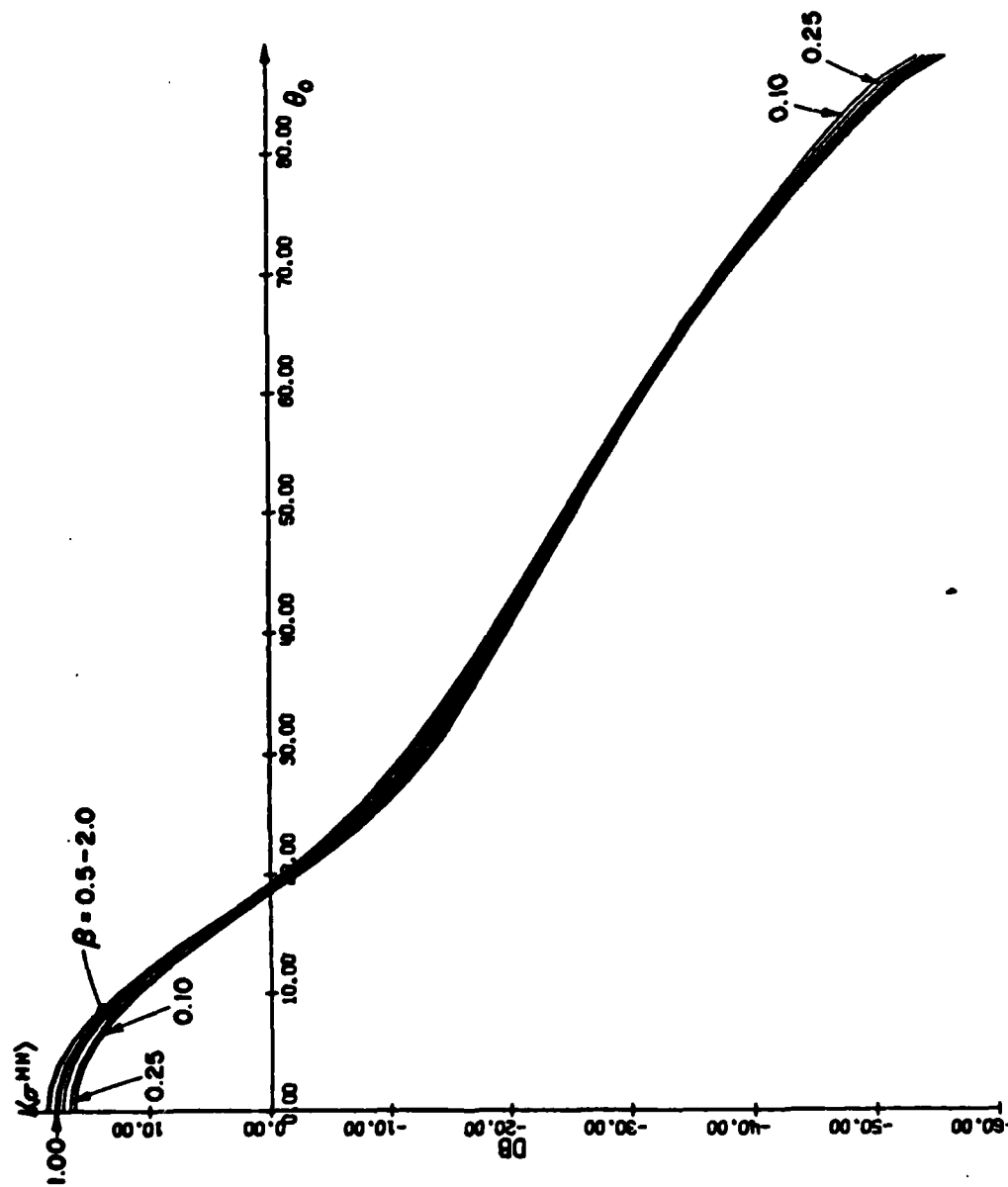


Fig. 3.11b. Backscatter cross sections for $\langle \sigma^{PQ} \rangle$ (total) for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.1, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75$, and $\beta = 2.0$. $\langle \sigma^{HH} \rangle$.

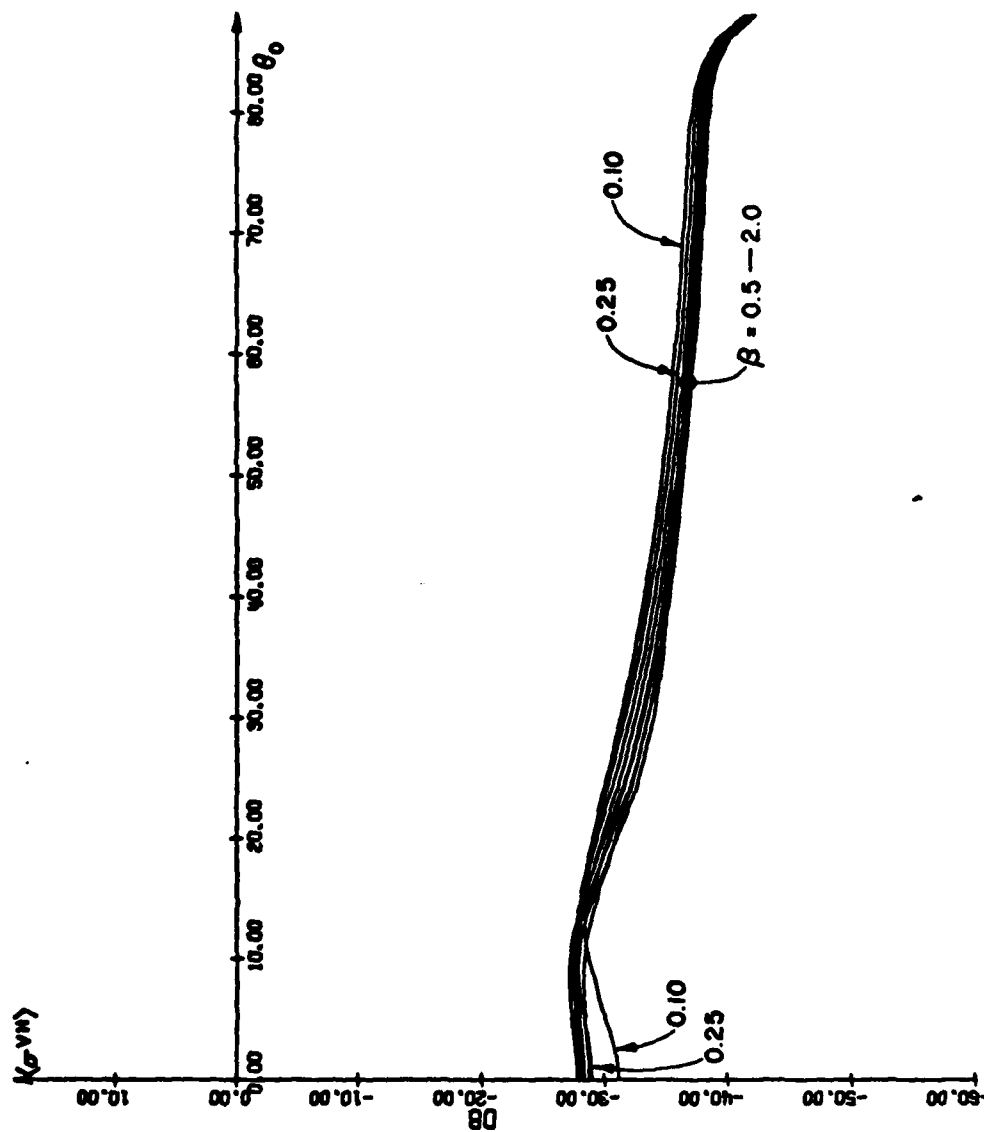


Fig. 3.11c. Backscatter cross sections $\langle \sigma^{PQ} \rangle$ (total) for rough surface (3.14) using full wave formulation (3.15) for $\beta = 0.1, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75$, and $\beta = 2.0, \langle \sigma^{VH} \rangle$.

Table 3.1

β	k_d	σ_{FS}^2
0.10	0.950	0.0223
0.25	0.602	0.0202
0.50	0.426	0.0187
0.75	0.348	0.0177
1.00	0.301	0.0171
1.25	0.269	0.0165
1.50	0.246	0.0161
1.75	0.228	0.0158
2.00	0.213	0.0155

Relationship between the parameter $\beta = 4k_o^2 \langle h_R^2 \rangle$, the surface wave number k_d (3.26) and the mean square slope σ_{FS}^2 (3.27)

k_d) and computational ease the optimal choice for β is 1.0. This corresponds to the choice of $k_d = 0.301 \text{ (cm}^{-1}\text{)}$ and $\lambda_d = 20.87 \text{ (cm)} = 10.44\lambda_o$. It should be pointed out that for $\beta = 1.0$ not only does $\langle \sigma^{PQ} \rangle_{R2}$ begin to become significant (particularly in the region between $\theta_o = 15^\circ$ and $\theta_o = 35^\circ$, where specular point scattering and Bragg scattering effects begin to blend), but the factors $|\chi^R(\bar{v} \cdot \bar{n}_g)|^2 = e^{-\beta}$ and $\exp(-v \frac{2}{y} \langle h_R^2 \rangle)$ are also significantly different from unity. These factors appearing in $\langle \sigma^{PQ} \rangle_F$ and $\langle \sigma^{PQ} \rangle_R$ respectively and the terms $\langle \sigma^{PQ} \rangle_{Rm}$ ($m \geq 2$) are not present in the expressions derived by Brown (1978). The contribution that the term $\langle \sigma^{PQ} \rangle_{R2}$ makes to the total backscatter cross section is even more pronounced for $P \neq Q$ since the physical optics approximation for the cross-polarized backscatter cross section $\langle \sigma^{PQ} \rangle_F$ is zero. Moreover, since $\langle \sigma^{PQ} \rangle_F = 0$ for $P \neq Q$, use of the two scale model of the composite surface to compute the cross-polarized cross sections needs to be investigated further (see Section 4).

The small though perhaps significant differences in the full wave values for $\langle \sigma^{PQ} \rangle$ (the total backscatter cross sections), as β increases from 0.1 to 1.0 are much smaller than those obtained from Brown's results. The reason why $\langle \sigma^{PQ} \rangle$ is sensitive to variations in β for $\beta \leq 1.0$ is that on deriving the approximate physical optics contribution $\langle \sigma^{PQ} \rangle_F$ (3.16) (associated with the filtered surface h_F) from the full wave solution (3.1), it was assumed implicitly in deriving $\langle \sigma_\infty^{PQ} \rangle_B$ (3.30) that the radii of curvature associated with the surface h_F satisfied the Kirchhoff approximations for the surface field and also that the individual specular points on the filtered rough surface h_F met the conditions for deep phase modulation; namely, that distances from the specular points on the surface h_F to the

receiver and transmitter were such that their contributions were distributed uniformly in phase between $-\pi$ and π . For $\beta < 1.0$ the neighboring specular points on h_F begin to merge and contribute more strongly to the backscatter cross section than when deep phase modulation applies.

For backscatter $\langle \sigma^{VH} \rangle = \langle \sigma^{HV} \rangle$, and therefore the numerical data for $\langle \sigma^{HV} \rangle$ is not presented. However, $\langle \sigma^{VV} \rangle$ is significantly different from $\langle \sigma^{HH} \rangle$ as $\theta_o \rightarrow 90$. This polarization dependence of $\langle \sigma^{PP} \rangle$ is missing in the physical optics results.

3.5 Concluding Remarks

In this paper, extensive numerical results for the full wave backscatter cross sections $\langle \sigma^{PQ} \rangle$ ($P, Q = V$ - vertical or H - horizontal) are presented. For convenience, and in order to compare the full wave results with earlier solutions to the scattering cross sections, the full wave solutions are expressed in terms of a weighted sum of a scattering cross section $\langle \sigma^{PQ} \rangle_F$, associated with the filtered surface h_F (consisting of the larger scale spectral components) and a remainder term $\langle \sigma^{PQ} \rangle_R$, associated with the surface \bar{h}_R (consisting of the smaller scale spectral components).

In an attempt to draw more definite conclusions about the choice of k_d (the wavenumber where spectral splitting between the surfaces h_F and \bar{h}_R is assumed to occur), the parameter $\beta = 4k_o^2 \langle \bar{h}_R^2 \rangle$ (that is related to k_d) is varied over a very wide range of values ($0.1 < \beta < 2.0$). It was not necessary to consider values of $\beta < 0.1$, since these results would practically duplicate those already provided by Brown (1978). For $\beta > 2.0$ the numerical evaluations become tedious and of no particular value. It is

shown that while there were small though significant variations in the values of $\langle \sigma^{PQ} \rangle$ as β increased from $\beta = 0.1$ to $\beta = 1.0$ the results merged for $\beta \geq 1.0$. Thus, from the point of view of computational ease and accuracy, the choice $\beta = 1.0$ corresponding to $\lambda_d = 10.44\lambda_0$ is optimal. It should be noted that in order to derive the physical optics - (specular point) contribution to the scattering cross section (associated with the filtered surface h_F) from the full wave solution, it was necessary to assume that the radii of curvature criteria (imposed by the Kirchhoff approximations for the surface fields), as well as the condition for deep phase modulation were satisfied. However, the parameter β is not restricted by the characteristics of the small scale surface. Thus β could be assumed to be much larger than 0.1 in gross violation of the standard perturbation criteria without affecting the results for the total scattering cross section. It is interesting to note that for $\beta = 2.0$ (Fig. 3.7) $\langle \sigma^{PQ} \rangle_F$ is significantly smaller than $\langle \sigma^{PQ} \rangle_R$ for all values of θ_0 including near normal incidence. Nevertheless, the computed results for $\langle \sigma^{PQ} \rangle$ with $\beta = 2.0$ (the total backscatter cross sections) are in agreement with the corresponding results for $\beta = 1.0$ where $\langle \sigma^{PQ} \rangle_F$ is the dominant term near normal incidence. This is because the full wave approach permits the blending of specular point scattering (from h_F consisting of the larger scale spectral components of the surface height) with Bragg scattering (from \bar{h}_R consisting of the smaller scale spectral components of the surface height). Thus, one cannot arbitrarily neglect the contribution to $\langle \sigma^{PQ} \rangle$ associated with the surface \bar{h}_R , even at near normal incidence. Furthermore, since the physical optics approximation for the cross-polarized backscatter cross section is zero, it is necessary to further

investigate the use of the two scale model of the rough surface to compute $\langle \sigma^{PQ} \rangle$, $P \neq Q$.

Although numerical results are presented in this paper only for backscatter, the full wave solution (3.1) is suitable for the evaluation of scattering in arbitrary directions. Furthermore, the medium $y < h(x,z)$ need not be perfectly conducting and the effects of finite conductivity can be considered. The full wave approach also accounts for the scattering of the lateral and surface waves (Bahar 1980a,b) that are excited over non-perfectly conducting surfaces. The full wave approach can be applied to scattering at low radio wave frequencies as well as at optical frequencies.

4.0 Computations of Rough Surface Cross Sections That Do Not Involve Spectral Splitting

4.1 Background

In Sections 2 and 3 the full wave solutions for the scattering cross sections were applied to two-scale models of composite rough surfaces. The main purpose of the work reported in these sections was to compare the full wave solutions for the like and cross polarized scattering cross sections with the solutions based on the use of a perturbed-physical optics approach. The question of the specification of k_d , the wavenumber where surface height spectral splitting is assumed to occur was also investigated in detail. It was shown that the full wave solution for the scattering cross sections can be expressed as a weighted sum of two cross sections. The first was associated with the filtered surface consisting of the large scale spectral components and the second was associated with the surface consisting of the small scale spectral components that ride on the large scale surface. Provided that the large scale surface satisfied the radii of curvature criteria (associated with the Kirchhoff approximations of the surface fields) and the condition for deep phase modulation, it was shown that the full wave solutions were insensitive to the variations in the specified value of k_d .

Since the full wave approach accounts for specular point scattering and Bragg scattering in a unified self consistent manner, it is not necessary to spectrally decompose the composite rough

surfaces into two surfaces with different roughness scales. Thus in this section the scattering cross sections for all angles of incidence are calculated using the undecomposed form of the full wave solution.

4.2 Discussion

On applying the full wave approach in Sections 2 and 3 to evaluate the like and cross polarized scattering cross sections for two scale models of composite rough surfaces, several assumptions were made to facilitate the computations. The first assumption was that the large and small scale surfaces were statistically independent (Brown 1978). It would seem reasonable to make such an assumption if the two surfaces are results of independent processes. This would be the case, for example, if the small scale roughness is due to erosion, while the large scale roughness is due to geophysical forces that result in hills and valleys, or as in the case of the sea, where the capillary waves are dependent on surface tension while the large scale rough surface is generated by gravity waves. For the general case, however, one cannot assume statistical independence of the large and small scale surfaces.

The second simplifying assumption that was made was that the mean square slope σ_s^2 for the total surface was approximately equal to the mean square slope σ_{FS}^2 for the filtered large scale surface.

The third assumption was that if the mean square height of the total rough surface is large compared to wavelength, the surface

height characteristic function for the total surface is negligibly small. The effects of these simplifying assumptions on the computed results for the cross sections are examined.

The physical optics approximation for the cross polarized backscatter cross section is zero. As a result, the cross polarized backscatter cross section for the filtered surface is set equal to zero when the two scale model is used. However, for backscatter only, the specular points on the rough surface do not depolarize the incident wave. Therefore, the justification for use of the two-scale model to evaluate the cross polarized scattering cross sections of composite rough surfaces is also examined in this section of the report.

4.3 Application of the Full Wave Solution without Surface

Decomposition

The starting point for this analysis is equation (3.1) for the like and cross polarized scattering cross sections of the rough surface $y = h(x, z)$

$$\begin{aligned} \langle \sigma^{PQ} \rangle = \frac{k_o^2}{\pi} \int_{-\infty}^{\infty} \langle S^{PQ} \exp[i v_y (h-h')] \rangle & - \left[\frac{D^{PQ} p_2(\bar{n}^f, \bar{n}^i | \bar{n})}{\bar{n} \cdot \bar{a}_y} \chi(v_y) \right]^2 \Bigg] \\ & \cdot \exp[i v_x x_d + i v_z z_d] dx_d dz_d \end{aligned} \quad (4.1)$$

in which

$$\bar{r}_d = (x-x')\bar{a}_y + (z-z')\bar{a}_z = x_d\bar{a}_x + z_d\bar{a}_z \quad (4.2)$$

is the radius vector between two points on the reference plane (x, z)

The vector \bar{v} is

$$\bar{v} = k_0 (\bar{n}^f - \bar{n}^i) = v_x \bar{a}_x + v_y \bar{a}_y + v_z \bar{a}_z \quad (4.3)$$

where k_0 is the free space wavenumber for the electromagnetic wave and \bar{n}^i and \bar{n}^f are unit vectors in the directions of the incident and scattered wave normals respectively. An $\exp(i\omega t)$ time dependence is assumed in this work. The symbol $\langle \rangle$ denotes the statistical average and

$$\begin{aligned} \frac{k_0^2}{\pi} \langle S^{PQ} \exp[i v_y (h - h')] \rangle &= \frac{k_0^2}{\pi} \int \left| \frac{D^{PQ}(F)}{\bar{n} \cdot \bar{a}_y} \right|^2 P_2(\bar{n}^f, \bar{n}^i | \bar{n}_s) p(h_x, h_z) dh_x dh_z \\ &\cdot \chi_2(v_y, -v_y) \equiv I^{PQ}(\bar{n}^f, \bar{n}^i) \chi_2(v_y, -v_y) \end{aligned} \quad (4.4)$$

in which

$\bar{n}(h_x, h_z)$ is the unit vector normal to the rough surface

$$f(x, y, z) = y - h(x, z) = 0 \quad (4.5)$$

Thus

$$\nabla f = \bar{n} |\nabla f| = \nabla(y - h(x, z)) = (-h_x \bar{a}_x + \bar{a}_y - h_z \bar{a}_z) \quad (4.6)$$

in which the components of the gradient of $h(x, z)$

$$h_x = \partial h / \partial x, \quad h_z = \partial h / \partial z \quad (4.7)$$

are random variables and $p(h_x, h_z)$ is the probability density function for the slopes h_x and h_z . The expression for the scattering cross sections $\langle \sigma^{PQ} \rangle$ (4.1) accounts for shadowing and

$$P_2(\bar{n}^f, \bar{n}^i | \bar{n}) = P_2(\bar{n}^f, \bar{n}^i, | \bar{n}_s) S(\bar{n}^f \cdot \bar{n}) S(-\bar{n}^i \cdot \bar{n}) \quad (4.8)$$

in which $P_2(\bar{n}^f, \bar{n}^i | \bar{n})$ is the probability that a point on the rough

surface is both illuminated and visible given the value of the slopes at the point (Smith 1967, Sancer 1968) and $P_2(\bar{n}^f, \bar{n}^i | \bar{n}_s)$ is its value at the specular points where the unit vector \bar{n} is given by

$$\bar{n} \rightarrow \bar{n}_s = \frac{\bar{n}^f - \bar{n}^i}{|\bar{n}^f - \bar{n}^i|} = \bar{v}/v \quad (4.9)$$

The arguments of the unit step functions $S(-\bar{n}^i \cdot \bar{n})$ and $S(\bar{n}^f \cdot \bar{n})$ vanish at points of the rough surface where the incident and scattered waves are tangent to the rough surface. Thus

$$S(-\bar{n}^i \cdot \bar{n}_s) = 1 \quad \text{and} \quad S(\bar{n}^f \cdot \bar{n}_s) = 1.$$

The characteristic and joint characteristic functions for the surface height h are respectively (Beckmann and Spizzichino 1963)

$$\chi(v_y) = \langle \exp(iv_y h) \rangle \quad (4.10)$$

and

$$\chi_2(v_y, -v_y) = \langle \exp[iv_y(h-h')] \rangle \quad (4.11)$$

It is assumed in this work that the probability density function for the surface height is jointly Gaussian. Thus

$$\chi(v_y) = \exp(-v_y^2 \langle h^2 \rangle) \quad (4.12)$$

and

$$\chi_2(v_y, -v_y) = \exp(-v_y^2 \langle h^2 \rangle - v_y^2 \langle hh' \rangle) \quad (4.13)$$

where $\langle h^2 \rangle$ is the mean square height and $\langle hh' \rangle$ is the surface height autocorrelation function. The coefficients D^{PQ} depend explicitly upon the polarization of the incident wave (second

superscript Q=V - vertical, Q=H - horizontal) and the polarization of the scattered wave (first superscript; P=V,H) the direction of the incident and scattered wave normals \bar{n}^i and \bar{n}^f respectively, the unit vector \bar{n} normal to the rough surface and the complex permeability and permittivity of the medium of propagation respectively (Bahar 1981a, see Section 1). On deriving (4.4) it is assumed that the rough surface is Gaussian and stationary, thus the surface height h and slopes (h_x, h_z) are statistically independent (Brown 1978, Longuet-Higgins 1957). It is also assumed that for distances $|\bar{r}_d|$ less than the surface height correlation distance, $\ell_c, \bar{n}(h_x, h_z) \approx \bar{n}'(h_x', h_z')$. It has been shown that if the principal contributions to the scattered fields come from specular points on the rough surface ($\bar{n} = \bar{n}_s$), (4.1) reduces to the physical optics solution for the scattering cross section. If, however, the roughness scale of the surface is small compared to the wavelength ($k_0^2 \langle h^2 \rangle \ll 1$) and the surface slopes h_x and h_z are very small, (4.1) reduces to the perturbation solution for the scattering cross sections (Rice 1951). Thus, in this case Bragg scattering is accounted for and the backscatter cross sections for grazing angles are strongly dependent on polarization. In Section 2 a two scale model is adopted to determine the corresponding full wave solution for the scattering cross sections. To facilitate the application of the two-scale model it is assumed that the small scale surface \bar{h}_R and the large scale filtered surface h_F

are statistically independent (Valenzuela 1968; Wright 1968; Brown 1978). This assumption is reasonable if the surfaces h_F and h_S are results of independent processes (Brown 1978) as for example, when the small scale roughness is due to erosion while the large scale roughness is due to geophysical forces that result in hills and valleys or as in the case of the sea, where the capillary waves are dependent on surface tension while the large scale surface is generated by gravity waves. In general, however, it cannot be assumed that the large and small scale roughness of the surface are statistically independent. In the general case, if the two scale model is used to analyze the problem it would be necessary to know the large and small scale surface height joint probability density function for two adjacent points on the rough surface to determine χ_2 (4.11) alone.

Since the full wave solutions account for both Bragg scatter and specular point scatter in a unified, self consistent manner, in this section solutions for (4.1) are developed without adopting a two-scale model of the rough surface.

In Sections 2 and 3 it has been noted that the physical optics approximation for the cross polarized backscatter cross section is zero ($\langle \sigma^{PQ} \rangle_F = 0$ for $P \neq Q$). However, even the large scale filtered surface will depolarize the backscattered field at non-specular points on the surface. Therefore the present analysis should shed more light on the evaluation of the like and cross

polarized backscatter cross sections and the suitability of the two scale model even if it can be assumed that the large and small scale surfaces are statistically independent.

Assuming that $k_0^2 \langle h^2 \rangle \ll 1$ and $|\chi|^2 \ll 1$, the scattering cross section (4.1) can be expressed as follows:

$$\begin{aligned} \langle \sigma^{PQ} \rangle &= I^{PQ}(\bar{n}^f, \bar{n}^i) \int_{-\infty}^{\infty} [\chi_2(v_y, -v_y) - |\chi(v_y)|^2] \\ &\quad \cdot \exp[i v_x x_d + i v_z z_d] dx_d dz_d \\ &\equiv I^{PQ}(\bar{n}^f, \bar{n}^i) Q(\bar{n}^f, \bar{n}^i, R) \end{aligned} \quad (4.14)$$

in which I^{PQ} is defined by (4.4) and Q , the two dimensional Fourier transform of $(\chi_2 - |\chi|^2)$ depends on the surface height correlation coefficient R

$$R \equiv \langle hh' \rangle / \langle h^2 \rangle. \quad (4.15)$$

Using the notation of Rice (1951), the surface height spectral density function $W(v_x, v_z)$ is related to the two dimensional Fourier transform of the surface height autocorrelation function.

$$W(v_x, v_z) = \frac{1}{\pi^2} \int \langle hh' \rangle \exp[i v_x x_d + i v_z z_d] dx_d dz_d \quad (4.16a)$$

and

$$\langle hh' \rangle = \int \frac{W(v_x, v_z)}{4} \exp[-i v_x x_d - i v_z z_d] dv_x dv_z \quad (4.16b)$$

Thus assuming that the rough surface is Gaussian and stationary, to compute the scattering cross sections (4.14) it is necessary to prescribe the two dimensional slope probability density function $p(h_x, h_z)$ (4.4) and the surface height autocorrelation function or

its Fourier transform (the surface height spectral density function).

Since it is assumed in this work that the surface is isotropic,

$\langle hh' \rangle$ depends only on the distance $r_d = |\vec{r}_d|$ between the two points (x, h, z) and (x', h', z') on the rough surface. Thus

$$\langle hh' \rangle = 2\pi \int \frac{W(v_{xz})}{4} J_0(v_{xz} r_d) v_{xz} dv_{xz} \quad (4.17)$$

and

$$\langle h^2 \rangle = 2\pi \int \frac{W(v_{xz})}{4} v_{xz}^2 dv_{xz} \quad (4.18)$$

in which J_0 is the Bessel function of order zero and

$$v_{xz}^2 = v_x^2 + v_z^2 \quad (4.19)$$

Since

$$J_0''(0) = [d^2 J_0(v_{xz} r_d) / dr_d^2]_{r_d=0} = v_{xz}^2 / 2 \quad (4.20)$$

$$R''(0) = \sigma_s^2 / 2 \langle h^2 \rangle = -\sigma_x^2 / \langle h^2 \rangle = -\sigma_z^2 / \langle h^2 \rangle \quad (4.21)$$

where

$$\sigma_s^2 = 2\pi \int \frac{W(v_{xz})}{4} v_{xz}^3 dv_{xz} \quad (4.22)$$

is the total mean square slope while σ_x^2 and σ_z^2 are the mean square slopes in the x and z directions. Thus for small values of r_d the correlation coefficient is given by (Beckmann and Spizzichino 1963, Brown 1978)

$$R(r_d) \approx 1 - r_d^2 / l_c^2 = 1 - \sigma_x^2 r_d^2 / 2 \langle h^2 \rangle \quad (4.23)$$

where l_c is the correlation distance.

4.4 Illustrative Examples

For the following illustrative examples the following special form of the surface height spectral density function is chosen

(Brown 1978)

$$W(v_x, v_z) = \left(\frac{2}{\pi}\right) S(v_x, v_z) = \begin{cases} \left(\frac{2}{\pi}\right) B k^4 / (k^2 + \kappa^2)^4 & k \leq k_c \\ 0 & k > k_c \end{cases} \quad (4.24)$$

where W is the spectral density function defined by Rice (1951) and S is the corresponding quantity used by Brown (1978). For the above isotropic model of the ocean surface

$$B = 0.0046 \quad (4.25a)$$

$$k^2 = v_x^2 + v_z^2 \text{ (cm)}^{-2}, \quad k_c = 12 \text{ (cm)}^{-1} \quad (4.25b)$$

$$\kappa = (335.2V^4)^{-1/2} \text{ (cm)}^{-1}, \quad V = 4.3 \text{ (m/s)} \quad (4.25c)$$

In (4.25c) V is the surface wind speed. The wavelength for the electromagnetic wave is

$$\lambda_0 = 2 \text{ (cm)}, \quad (k_0 = 3.1416 \text{ (cm)}^{-1}) \quad (4.26)$$

Substituting (2.24) into equation (4.18) for the mean square height of the rough surface yields

$$\begin{aligned} \langle h^2 \rangle &= \frac{B}{2} \left[\frac{1}{3\kappa^2} - \frac{1}{k_c^2 + \kappa^2} + \frac{\kappa^2}{(k_c^2 + \kappa^2)^2} + \frac{\kappa^4}{3(k_c^2 + \kappa^2)^3} \right] \\ &= \frac{B k_c^6}{6\kappa^2 (\kappa^2 + k_c^2)^3} \end{aligned} \quad (4.27)$$

Thus if the spectral cut-off point k_c (Brown 1978) is much larger than κ (as for the illustrative example (4.25))

$$\langle h^2 \rangle \approx \frac{B}{6\kappa^2} \quad (4.28)$$

The surface height autocorrelation function $\langle hh' \rangle$ (4.17) can be expressed in closed form for $k_c \rightarrow \infty$. Thus the surface height correlation coefficient $R(r_d)$ (4.15) is given by (Miller et al 1972)

$$R(r_d) = \left[1 + \frac{1}{8} (\kappa r_d)^2 \right] (\kappa r_d) K_1(\kappa r_d) - (\kappa r_d)^2 K_0(\kappa r_d) \quad (4.29)$$

in which K_0 and K_1 are the modified Bessel functions of the second kind and of order zero and one respectively. Since $k_c \gg K$ and $k_c > k_0$ the above closed form expression is used for R in this illustrative example. The total mean square slope of the rough surface is obtained on substituting (4.24) into (4.22).

$$\begin{aligned} \sigma_s^2 &= B \left[\frac{1}{2} \ln \frac{k_c^2 + \kappa^2}{\kappa^2} - \frac{11}{12} + \frac{3}{2} \frac{\kappa^2}{k_c^2 + \kappa^2} - \frac{3}{4} \frac{\kappa^4}{(k_c^2 + \kappa^2)^2} + \frac{1}{6} \frac{\kappa^6}{(k_c^2 + \kappa^2)^3} \right] \\ &= B \left[\frac{1}{2} \ln \frac{k_c^2 + \kappa^2}{\kappa^2} - \frac{k_c^2 (6\kappa^2 + 15\kappa^2 k_c^2 + 11k_c^4)}{12(\kappa^2 + k_c^2)^3} \right] \end{aligned} \quad (4.30)$$

Thus for $k_c \gg \kappa$

$$\sigma_s^2 = B \left[\frac{1}{2} \ln \frac{k_c^2 + \kappa^2}{\kappa^2} - \frac{11}{12} \right] \quad (4.31)$$

For typical sea surfaces the relative complex dielectric coefficient at 1.5 GHz is given by (Stogryn 1971)

$$\epsilon_r = 42 - i39$$

The slope probability density function $p(h_x, h_z)$ is assumed to be Gaussian, thus

$$p(h_x, h_z) = \frac{1}{\pi \sigma_s^2} \exp \left[- \frac{h_x^2 + h_z^2}{\sigma_s^2} \right]$$

In Fig. 4.1 the like polarized backscatter cross section $\langle \sigma^{VV} \rangle$ is plotted as a function of the angle of incidence θ_0^1 using the expression derived in Section (4.3). These results are compared with the two scale full wave results (Sections 2 and 3) based on the choice of k_d (the wavenumber where spectral splitting occurs) corresponding to $\beta = 1$. (see Section 2 equations (2.32a), (2.32b) and (2.32c)). Both results yield the same general dependence of $\langle \sigma^{VV} \rangle$ on the angle of incidence. The small difference in level is primarily due to the fact that in (4.3) the mean square slope σ_s^2 of the total (unfiltered) surface is used, (4.31), while in (2.32) the mean square slope σ_{FS}^2 for the filtered surface h_F is used (3.27). It should be noted that in deriving the expressions for the scattering cross sections based on the two-scale model (2.32), it was assumed that $\sigma_{FS}^2 \approx \sigma_s^2$. Thus the results based on (4.3) are more accurate. Furthermore, on deriving (2.32), using the two-scale model, the quantity $\chi(v_y)$ (4.12) is assumed to be negligible compared to $\chi_2(v_y, -v_y)$ (4.13) for $r_d \ll \ell_c$. Since $4k_0 \langle h^2 \rangle = 3468$ for this illustrative example, the resulting approximation is very good except very near grazing angles. In Figure 4.2 the corresponding results are given for the horizontally polarized backscatter cross sections $\langle \sigma^{HH} \rangle$. It is interesting to note that the full wave solution (4.3) yields the proper polarization dependence of the scattering cross sections for all angles of incidence without use of a two scale model since it accounts for specular point and Bragg scattering in a

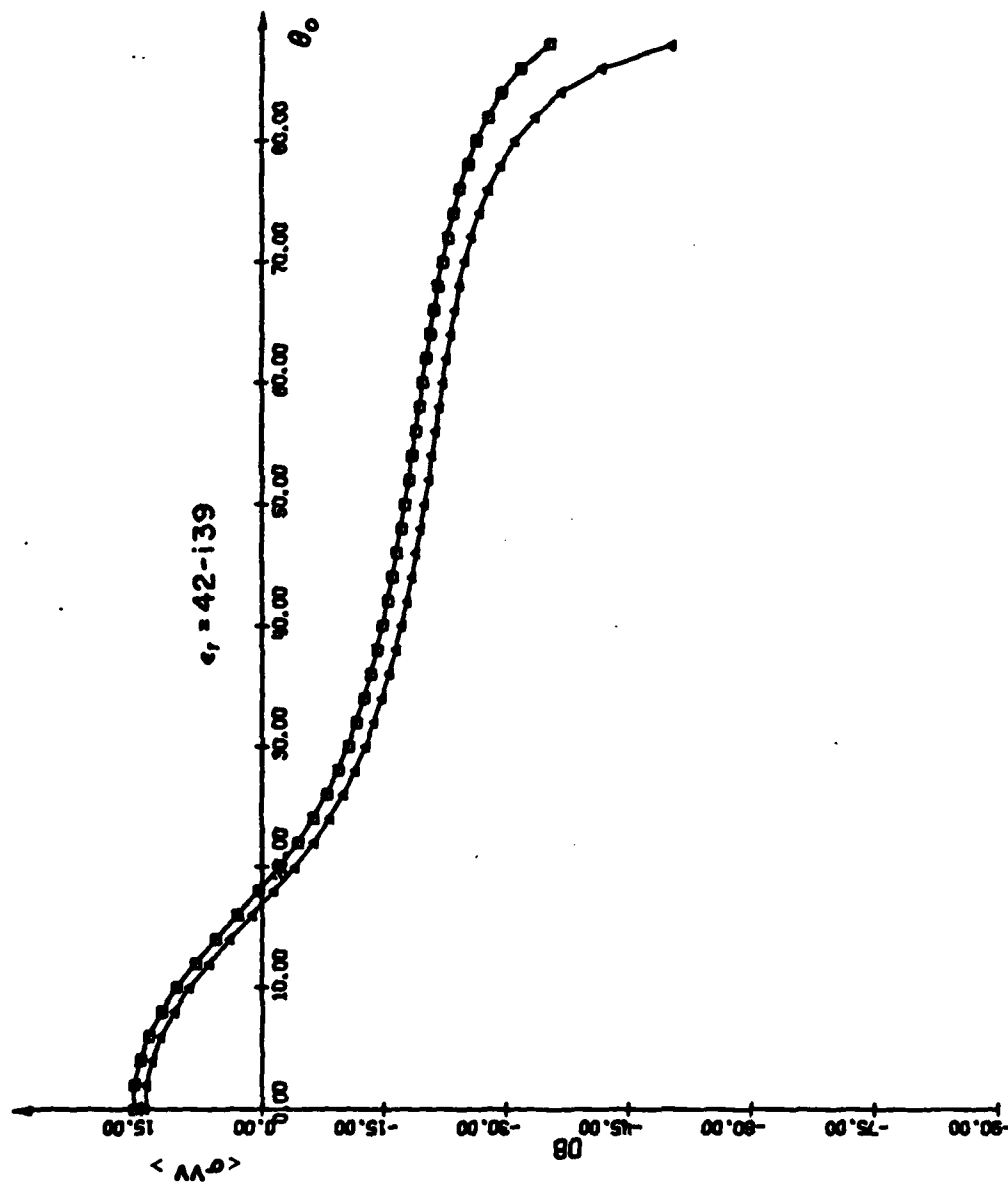


Fig. 4.1. Backscatter cross sections $\langle \sigma_{VV} \rangle$ for rough surfaces characterized by $W(V_x, V_z)$ given by (4.24).
 (□) Two scale model (2.32) and (Δ) unified full wave solution (4.4). Relative complex permittivity is $\epsilon_r = 42-i39$.

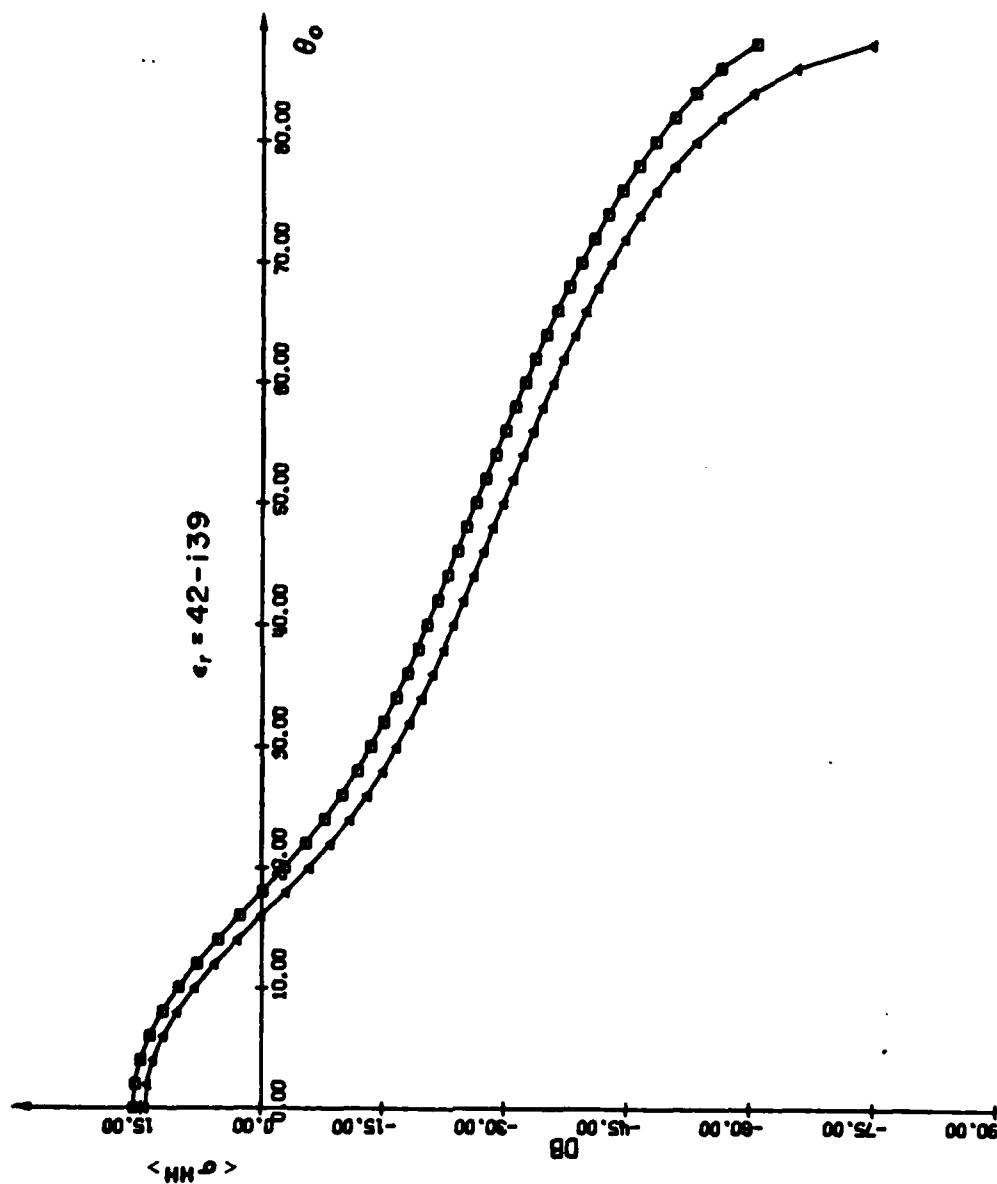


Fig. 4.2. Backscatter cross sections $\langle \sigma_{HH} \rangle$ for rough surfaces characterized by $W(V_x, V_z)$ given by (4.24).

(\square) Two scale model (2.32) and (Δ) unified full wave solution (4.4). Relative complex permittivity is $\epsilon_r = 42 - i39$.

unified, self consistent manner. In Figure 4.3 the cross polarized backscatter cross sections $\langle \sigma^{VH} \rangle = \langle \sigma^{HV} \rangle$ are plotted as functions of the angle of incidence. Here too, both the solutions based on the two scale model as well as the solution derived in this section are presented. Unlike the solutions for the like polarized backscatter cross sections $\langle \sigma^{PP} \rangle$ ($P=V,H$), the solutions for the cross polarized backscatter cross sections differ significantly, especially near normal incidence where the difference in level is about 15db. This very significant difference is due to the fact that the physical optics approximations for the cross polarized backscatter cross section is zero (Brown 1978, see Section 2). For backscatter the surface at the specular points is normal to the incident wave. At these stationary phase points no depolarization occurs. However, since depolarization occurs at the non specular points of the filtered surface, the physical optics approximations for the cross polarized backscatter cross section is not valid. It is interesting to note that for the two scale model at normal incidence

$$\langle \sigma^{PP} \rangle / \langle \sigma^{PQ} \rangle \approx 47\text{db} \quad (P \neq Q)$$

However, using the full wave solution (4.3)

$$\langle \sigma^{PP} \rangle / \langle \sigma^{PQ} \rangle \approx 31\text{db} \quad (P \neq Q)$$

The latter results are significantly more in line with published experimental results* (Long 1975).

*See also NRL report on "Airborne Radar Backscatter Study at four frequencies", NRL Prob ROZ-37, SER:8560, August 1966, by J.C.Daley

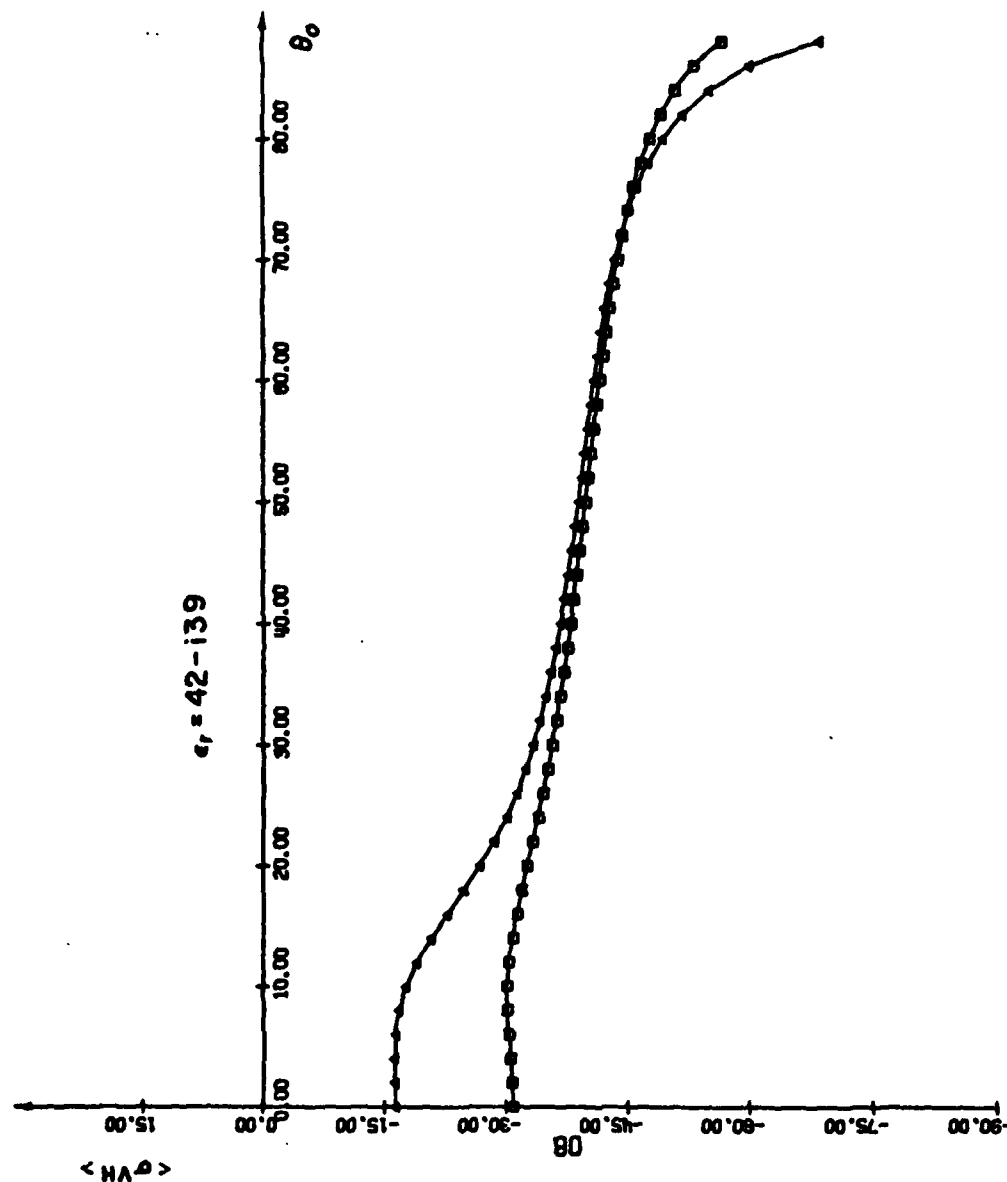


Fig. 4.3. Backscatter cross sections $\langle \sigma^{VH} \rangle = \langle \sigma^{HV} \rangle$ for rough surfaces characterized by $W(V_x, V_z)$ given by (4.24). (\square) Two scale model (2.32) and (Δ) unified full wave solution (4.4). Relative complex permittivity is $\epsilon_r = 42 - i39$.

4.5 Concluding Remarks

It is shown in this section that two-scale models of rough surfaces can be adopted to obtain solutions for the like polarized backscatter cross sections that are in reasonably good agreement with the full wave solutions derived in this section. However, the two-scale model cannot be used to evaluate the cross polarized backscatter cross sections. The significant differences between the solutions derived in this section and those based on the two scale models (Section 2 and 3) are primarily due to the fact that the physical optics approximation for the cross polarized backscatter cross section (associated with the large scale filtered surface) is zero. For backscatter, the specular points lie on portions of the rough surfaces that are perpendicular to the incident wave normal \vec{n}^i ($\vec{n}_s = \vec{n}^f = -\vec{n}^i$). At these specular points, the backscattered waves are not depolarized. However at non-specular points of the rough surface, the backscattered waves are depolarized (Bahar 1981b). Thus it is important to note that even if a surface satisfies the radii of curvature criteria (associated with the Kirchhoff approximations for the surface fields), the physical optics approximations for the scattered fields may not be valid unless for the given incident and scatter angles specular points exist on the surface and significant contributions to the scattered fields come from these stationary phase-specular-points of the surface. This explains why the physical optics approximations for the like polarized backscattered cross sections are not suitable for grazing angles even if the surface meets the radii of curvature criteria associated with the Kirchhoff approximations.

There are additional important reasons for preferring to use the analysis developed in this section over those that are based on two-scale models of rough surfaces. Firstly, if the two-scale model is used, it is necessary to assume that the large and small scale surfaces are statistically independent (Brown 1978). Secondly, even if the assumption of statistical independence is acceptable, when the two-scale model is used, it is still necessary to judiciously specify k_d (where spectral splitting is assumed to occur). These problems do not arise when the unified full wave formulation is used to evaluate the scattering cross sections.

While the preliminary results reported in this section are very significant in that they explain why the earlier solutions based on two-scale models of composite rough surfaces cannot be used to derive the cross polarized backscatter cross sections, more work needs to be done to apply the full wave solutions to more general (non-Gaussian, anisotropic) models of rough surfaces.

5.0 Concluding Remarks - Summary of Research During the Next Six Months - Proposed Future Investigations.

During the past two years the following principal contributions have been made with regard to the application of the full wave approach to rough surface scattering.

(a) The stationary phase approximation of the full wave solution is precisely equal to the physical optics solution for the scattered fields. Thus the full wave solution reduces to the physical optics solution when the expression for the local normal to the rough surface is replaced by its value at the stationary phase (specular) points on the surface. The questions surrounding the different existing forms of the physical optics solutions and the associated "edge term" have also been resolved.

(b) The full wave solution reduces to the geometrical optics approximations when the surface integrals are evaluated analytically using the steepest descent method.

(c) The small scale surface roughness and small slope approximation of the full wave solution is precisely equal to the perturbation solution for rough surface scattering. Thus the full wave solution reduces to the perturbation solution when the expression for the local normal to the rough surface is replaced by the normal to the reference plane.

(d) Apparent discrepancies between the physical optics and perturbation theories have been resolved. Both theories are sound if they are strictly applied to surfaces that satisfy the respective assumed restrictions. Physical optics is applicable if the principal

contributions to the scattered fields come from specular (stationary phase) points on the rough surface. Thus in order to apply the physical optics approach to rough surface scattering it is not sufficient to satisfy the radii of curvature criteria associated with the Kirchhoff approximations for the surface fields.

(e) The full wave approach provides the proper limiting forms for the far fields scattered at near grazing angles. It is shown that as the incident and scattered fields approach grazing angles the plane or spherical wave approximations for the far field are not suitable. In this case on evaluating the integrals for the far fields (in the wavenumber space) it is necessary to account for the fact that for grazing incident and scatter angles, poles are located in the vicinity of the saddle points. Thus it is shown that for grazing angles the range dependence of the fields is given by the error function complement and the full wave solutions exhibit the proper transition in the neighborhood of the shadow region.

(f) The full wave approach was applied to two-scale models of rough surfaces and compared with earlier solutions based on the composite, two-scale description of rough surfaces. The full wave solutions are expressed as a weighted sum of two cross sections, the first associated with the filtered surface consisting of the large scale spectral components of the rough surface and the second associated with the surface consisting of the small scale spectral components. Thus both specular and Bragg scattering are accounted for. The weighting function multiplying the physical optics cross section associated with the filtered surface accounts for the degradation of the cross section in the

specular direction due to the presence of the small scale surface that rides on the filtered surface. It is shown that if the specification of the wavenumber k_d (where spectral splitting is assumed to occur) can be based solely on the characteristics of the small scale surface and if the mean square slopes of the rough surface are very small, the full wave solution is in agreement with solutions based on a perturbed physical optics approach (Brown, 1978) or are based solely on physical considerations (Wright, 1968; Valenzuela, 1968). If the mean square slopes are not very small, Valenzuela's solution is not in agreement with Brown's solution. It is shown through the use of the full wave approach, that the difference is due to the fact that in Brown's work the correlation distance for the small scale surface is measured in the reference plane rather than along the large scale surface as in Valenzuela's work. Furthermore, in Brown's work the small scale surface height is measured perpendicular to the reference plane rather than normal to the filtered, large scale surface. This is contrary to Burrows' perturbation theory upon which Brown's solution is based.

(g) The controversy between Brown on the one hand and Hagfors and Tyler on the other regarding the specification of the wavenumber k_d where spectral splitting is assumed to occur is also resolved through the use of the full wave approach. As noted above, Brown states that the specification of k_d must be based solely on the characteristics of the small scale surface while Hagfors and Tyler specify k_d on the basis of the characteristics of the large scale

surface. Since the full wave approach accounts for specular point and Bragg scattering in a uniform self consistent manner, k_d may be specified arbitrarily. Provided that the filtered large scale surface satisfies both the radii of curvature criterion as well as the condition for deep phase modulation, the scattering cross sections based on the full wave approach are shown to be insensitive to the specific choice of k_d . It is shown that if one uses the two-scale model rough surfaces it is judicious to specify k_d such that $\beta = 4k_0^2 \langle h_s^2 \rangle = 1$. However since the physical optics approximation for the cross polarized backscattered cross sections are zero, the two-scale model can only be used to evaluate the like polarized backscattered cross sections.

(h) A unified full wave approach has been developed to obtain the scattering cross sections. Since this unified approach does not adopt a two-scale model of the rough surface, it does not artificially separate specular point scattering from Bragg scattering. Furthermore, using this uniform approach the question surrounding the specification of the wavenumber k_d does not arise nor is it necessary to assume that the large and small scale surfaces are statistically independent. It is shown using this approach that the two-scale model may be adopted to evaluate the like polarized cross sections, but that it should not be used to evaluate the cross polarized backscatter cross sections. The physical optics cross section associated with the filtered surface vanishes for backscatter because the specular points are located on regions of the surface that are perpendicular

to the incident wave. Thus the backscattered fields are not depolarized from specular points. For backscatter, depolarization is due to scattering from non-specular points. Therefore, as shown through the use of the full wave approach, the backscattered fields are depolarized even by the large scale filtered surface.

During the next six months (the final term of the current contract) the following phases of our research will be developed further:

- (a) Extension of the preliminary investigation using the unified full wave approach (see Section 4).
- (b) Investigation of the dependence of the scattering cross section upon the complex permittivity characterizing the rough surfaces.
- (c) Research reported in the Interim Technical Report on "Scattering Cross Sections for Composite Models of Non-Gaussian Rough Surfaces for Which Decorrelation Implies Statistical Independence" revised and re-submitted for publication in scientific/technical journals.
- (d) Computer programs updated to reflect recent advances in the analytical-numerical work. A detailed listing of the computer programs will be presented in the final report. The documentation will also include flow charts and relevant comments for the convenience of the user.

It is proposed that the following topics be considered for future investigation as an extension of the current contract:

- (a) Scattering and depolarization due to rough surfaces covered by vegetation
- (b) Scattering due to variations in the complex permittivity, $\epsilon(\vec{r})$, that characterizes the rough surface (mixed path propagation using the full wave approach)

- (c) Application to synthetic aperture radars
- (d) The dependence of the scattering cross sections on different rough surface height spectral density functions
- (e) Extension of the research on scattering by non-Gaussian rough surfaces
- (f) Application of the full wave approach to anisotropic rough surfaces.

The extension of the current research could be conducted over a 30-month period at approximately the current level of support. The principal investigator will consider additional applications that are of interest to the contractor.

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